California's Common Core *Mathematics Framework*

California Mathematics Council – South
54th Annual Mathematics Conference
November 1-2, 2013 Palm Springs

CALIFORNIA DEPARTMENT OF EDUCATION
Tom Torlakson, State Superintendent of Public Instruction

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California's Common Core *Mathematics Framework*

Presenters:

- Dr. Christopher Yakes, CSU Chico
- Ed D'Souza, Instructional Quality Commission
- Julie Spykerman, Instructional Quality Commission
- Tom Adams, California Department of Education
- Dr. Jane Liang, California Department of Education
- Deborah Franklin, California Department of Education

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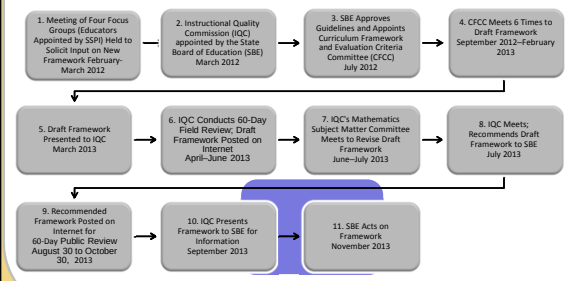
Mathematics Framework
Development Process

This chart shows the major steps of the curriculum framework development process.

All meetings are open to the public.



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Goals for the *Mathematics Framework*

- Guide the field in implementing the CA CCSSM
- Emphasize coherence across and within grade levels
- Integrate the Mathematical Practice and Content Standards
- Provide guidance on the higher mathematics course progression

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What is in the *Mathematics Framework*?

- Introduction
- Overview of Standards Chapters
- Grade-level chapters, TK–8
- Higher mathematics chapters by course:
  - Traditional pathway
  - Integrated pathway
  - Pre-calculus, Statistics and Probability
  - Advanced Placement Probability and Statistics
  - Calculus
  - Mathematical Modeling

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What is in the *Mathematics Framework*?

- Universal Access
- Instructional Strategies
- Supporting High-Quality Common Core Mathematics Instruction
- Technology in the Teaching of Mathematics
- Assessment
- Instructional Materials to Support the CA CCSSM (including the evaluation criteria for the mathematics adoption)

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What is in the Appendix?

1. Course Placement and Sequences
2. Financial Literacy and Mathematics Education
3. Possible Adaptations for Students with Learning Difficulties in Mathematics
4. Mathematical Modeling
5. Higher Mathematics Pathways Standards Chart
6. Method Used for Solving Single-digit Addition and Subtraction Problems

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What Guided the Revision of the *Mathematics Framework*?

- National documents and research from the Common Core State Standards Initiative
- Achieve the Core and the Progressions Documents
- The Standards for Mathematical Practice
- State Board of Education Guidelines

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SBE Guidelines for the Revision of the *Mathematics Framework*

- Based on input from the focus group meetings, written comments received, and statutory requirements
- Reviewed and recommended by the IQC and approved by the SBE
- The Mathematics CFCC members develop a framework based on the guidelines

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Who contributed to the draft *Mathematics Framework*?

- Focus Group members—all educators in California K–12 public schools, four regional meetings
- MCFCC members—one-half teachers, including teachers with experience teaching English learners and students with disabilities, other educators, and two content experts with Ph.Ds. in mathematics
- IQC—teachers, curriculum leaders, and administrators
- Staff of the Curriculum Frameworks and Instructional Resources Division and mathematics expert Dr. Christopher Yakes

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Who contributed to the draft *Mathematics Framework*?

- The field—provided comments on the draft framework during two 60-day review periods
- County Offices of Education—held discussion forums on the 1st draft of the *Mathematics Framework*
- Common Core State Standards for Mathematics author and expert Jason Zimba
- WestEd's California Comprehensive Center, Neal Finkelstein and Dona Meinders
- Staff of the California Department of Education's Language Policy and Leadership Office, STEM Office, and Assessment Transition Office

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Content of the *Mathematics Framework,* K–5

Focus Coherence Rigor

- A focus on understanding addition, subtraction, multiplication, and division (the four operations)
- Building from whole numbers in K–2 to fractions in grades 3–5
- Expectations of fluency with whole numbers and fractions

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Some K–5 examples:

WHAT STUDENTS LEARN IN GRADE ONE

Grade One Critical Areas of Instruction

- In grade one instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of and composing and decomposing geometric shapes. (excerpt from Grade 1)

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Some K–5 examples:

Focus, Coherence, and Rigor:

- As students use various counting strategies when they participate in counting activities they reinforce their understanding of the relationship between numbers and quantities and support mathematical practices such as modeling with mathematics (**MP.4**), the use of precise language (**MP.6**), and repeated reasoning to find a solution (**MP.8**). (excerpt from Kindergarten)

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Some K–5 examples:

Focus, Coherence, and Rigor

- Students' work with concepts of angle measures (**4.MD.5a and 7**) also connects to and supports adding fractions, which is major work at the grade in the cluster "Building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers" (**4.NF.3-4▲**). For example, a one degree measure is a fraction of an entire rotation and adding angle measures together is the same as adding fractions with a denominator of 360. (excerpt from Grade 4)

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Some K–5 examples:

Common Misconceptions:

- Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they think that $0.2 < 0.007$ simply because $2 < 7$.
- Students sometimes think the longer the decimal number the greater the value. For example, they think that 0.03 is greater than 0.3. (excerpt from Grade 4)

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Content of the *Mathematics Framework,* 6–8

Focus Coherence Rigor

- A focus on ratio, rates, percent, and statistics and probability
- Extending operations with fractions to rational numbers
- Expectations of fluency with expressions and linear equations

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Some 6–8 examples:

WHAT STUDENTS LEARN IN GRADE EIGHT

Grade Eight Critical Areas of Instruction

In grade eight, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence and understanding and applying the Pythagorean Theorem. (excerpt from Grade 8)

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Some 6–8 examples:

Focus, Coherence, and Rigor

Proportional relationship problems support mathematical practices as students make sense of problems (**MP.1**), reason abstractly and quantitatively (**MP.2**), and model proportional relationships (**MP.4**). For example, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building for modeling purposes. (excerpt from Grade 7)

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Some 6–8 examples:

Common Misconceptions:

Students may confuse dividing a quantity **by ½** with dividing a quantity **in half**. Dividing **by ½** is finding how many ½ sized portions there are, as in "dividing 7 **by ½**" which is 7 ÷ ½ = 14. On the other hand, to divide a quantity **in half** is to divide the quantity into two parts equally, as in "dividing 7 **in half**" yields 7/2 = 3.5. Students should understand that dividing **in half** is the same as dividing by two. (excerpt from Grade 6)

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Content of the *Mathematics Framework,* Higher Mathematics

- Traditional Pathway (Algebra I, Geometry, Algebra II)
- Integrated Pathway (Mathematics I, II, and III)
- Precalculus
- Statistics and Probability
- Calculus
- AP Probability and Statistics
- Mathematical Modeling

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Content of the *Mathematics Framework,* Higher Mathematics

What Students Learn in Mathematics I

In Mathematics I, students continue their work with expressions and modeling and analyzing situations. In earlier grades, students informally define, evaluate, and compare functions, and use them to model relationships between quantities. In Mathematics I, students will learn function notation and develop the concepts of domain and range. (excerpt from Mathematics I)

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Content of the *Mathematics Framework,* Higher Mathematics

Examples of Key Advances from Previous Grades or Courses

Themes from middle school algebra continue and deepen during high school. As early as grade 6, students began thinking about solving equations as a process of reasoning (6.EE.5). This perspective continues throughout Algebra I and Algebra II (A-REI).4 "Reasoned solving" plays a role in Algebra II because the equations students encounter can have extraneous solutions (A-REI.2). (excerpt from Algebra II)

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Content of the *Mathematics Framework,* Higher Mathematics

Focus Coherence Rigor

- A focus on the mathematics that students need for success in college and careers
- Extending from algebraic concepts to calculus, trigonometry, and advanced statistics
- Expectation that students are college and career ready and able to utilize mathematics in their lives

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Content of the *Mathematics Framework,* Higher Mathematics

MP.6. Attend to Precision.

Students begin to understand that a *rational number* has a specific definition, and that *irrational numbers* exist. They make use of the definition of *function* when deciding if an equation can describe a function by asking, "Does every input value have exactly one output value?" (excerpt from Mathematics II)

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Content of the *Mathematics Framework,* Higher Mathematics

Throughout the Mathematics II chapter, the examples given will be framed as much as possible as modeling situations, to serve as illustrations of the concept of mathematical modeling. The big ideas of quadratic functions, graphing, solving equations, and rates of change will be explored through this lens. (excerpt from Mathematics II)

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Course Sequences for Higher Mathematics

SBE Guidelines state: include a "discussion of options for middle school acceleration to support Algebra I or Integrated Mathematics I prior to ninth grade that are consistent with other Common Core states."

Acceleration decision points at middle school—between sixth and seventh grade—and in high school, after grade eight

- Acceleration in middle school
- Doubling up, enhanced pathway, or summer bridge in high school

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Course Sequences for Higher Mathematics: No Acceleration

Grade 6 → Grade 7 → Grade 8 → Algebra I /Math I → Geometry /Math II → Algebra II /Math III → Precalculus

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Possible Course Progressions from the Standards Document

Discipline	Grade Seven	Grade Eight	Grade Nine	Grade Ten	Grade Eleven	Grade Twelve
Algebra I/Mathematics I						
Geometry/Mathematics II						
Algebra II/Mathematics III						
Advanced Placement Probability and Statistics						
Calculus						

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Course Sequences for Higher Mathematics: Middle School Acceleration

Grade 6 → Grade 7 + Part of Grade 8 → Part of Grade 8 + Algebra I or Mathematics I → Geometry or Mathematics II → Algebra II or Mathematics III → Precalculus → Calculus

Acceleration Decision Point

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Course Sequences for Higher Mathematics: Doubling Up

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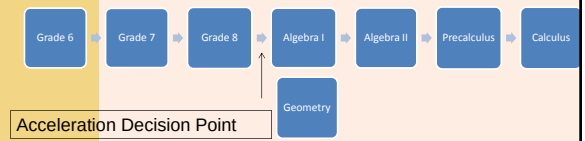
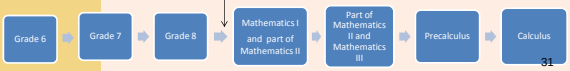
Doubling up in High School



Acceleration Decision Point

Accelerated Integrated Pathway



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Course Sequences for Higher Mathematics: Enhanced & Summer Bridge

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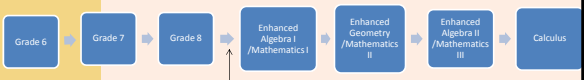
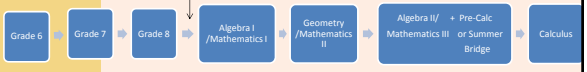
Enhanced Pathway



Acceleration Decision Point

Summer Bridge Pathway



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Algebra I Graduation Requirement

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Education Code 51224.5.

(b) Commencing with the 2003–04 school year and each year thereafter, at least one course, or a combination of the two courses, in mathematics required to be completed pursuant to subparagraph (B) of paragraph (1) of subdivision (a) of Section 51225.3 by pupils while in grades 9 to 12, inclusive, prior to receiving a diploma of graduation from high school, shall meet or exceed the rigor of the content standards for Algebra I, as adopted by the State Board of Education pursuant to Section 60605.

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Assessment Update

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2014 Mathematics Instructional Materials Adoption

- Evaluation Criteria
  - Approved by the SBE January 2013
- Programs
  - K–8 and Algebra I/Mathematics I
  - 35 program submissions
  - 30 recommended for adoption by the Review Panels
- Over 100 IMRs and CREs participated

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2014 Mathematics Instructional Materials Adoption

Next Steps:

- **November 21–22, 2013:** Instructional Quality Commission acts on adoption recommendations from the Review Panels
- **January 15 –16, 2014:** State Board acts on adoption recommendations from the Instructional Quality Commission

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Questions?

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Here's How to Keep Up-to-Date on Common Core Implementation and Smarter Balanced

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Subscribe to the Smarter Balanced Listserv:

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Questions on the *Mathematics Framework*?

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