Problem Solving with Core Content

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Session #345 Friday 1:30 -3:00, PSCC Mesquite D
2013 Palm Springs Mathematics Conference
California Mathematics Council – South

Abstract:

Problem solving and content mastery should reinforce one another rather than be an either/or proposition. We discuss how to apply CCSS Mathematical Practices through accessible problem solving experiences in a content-rich setting. We show how to invigorate lessons with non-routine problems that stretch content mastery and foster creativity, critical thinking, and perseverance. Emphasis is on multi-step problems in which students must make decisions to bring together the steps to construct solutions. The critical thinking is deliberately small in scope, with manageable decision making set against the backdrop of problems rich in mathematical content. Plenty of examples in a middle grades context will be worked through and discussed. The talk extends themes from the speaker's cover article "No Child Left Unchallenged" in the November 2011 issue of *Mathematics Teaching in the Middle school*. The talk is given in a middle grades context, but many of the ideas will also be of value for both elementary school and high school teachers. Print copies of handouts and samples will be provided during the talk.

Three Examples From Talk:

The talk includes several examples: some accompanied by student work and others to be worked through together during the talk. Here are three examples that focus on Practice Standard 1: *Make sense of problems and persevere in solving them.*

1. Go Beyond Methodology

> **Mean and Median Comparison:** Find three numbers such that their mean is 10 times as large as their median. Explain your reasoning.

Good problems should go beyond simply asking students to faithfully apply a learned algorithm or methodology. Instead of computing mean and median from a given set of numbers, students are asked to make decisions and create a set of numbers with a specified relationship between mean and median. Amanda's solution in **figure 1** shows a guess-and-check approach that leads to increasing student awareness that the third number in an ordered list must be much larger than the smallest two numbers to obtain a mean that is 10 times the median. She achieves a deeper understanding of mean and median by searching for a target comparison than by simply computing the quantities from a given set of numbers.

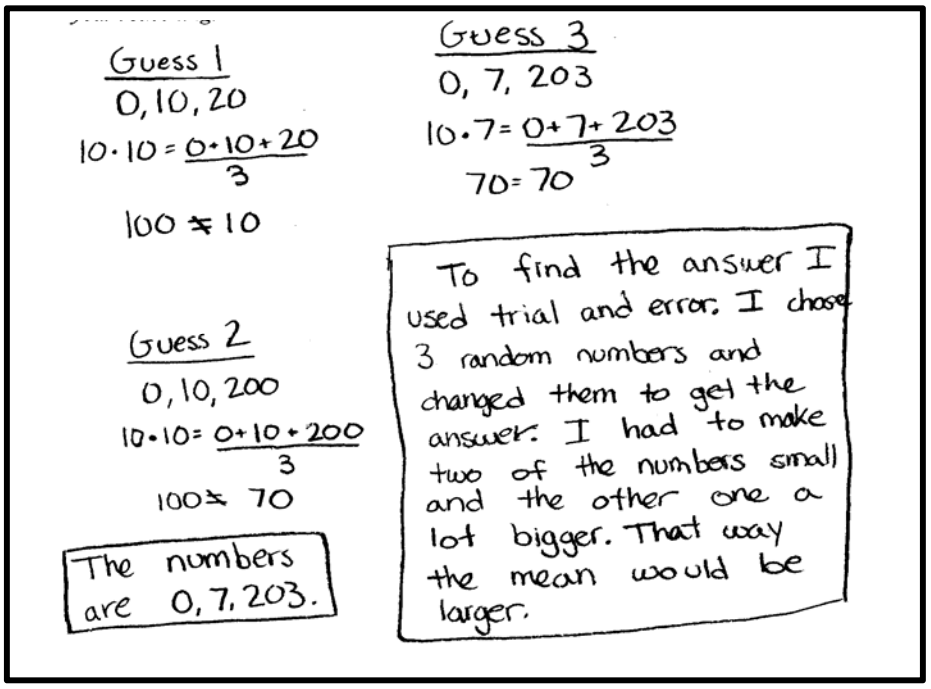

Figure 1. Amanda's solution to the Mean and Median Comparison problem.

Alex shows elementary algebra to frame his guess-and-check approach in **figure 2**, using variables to construct an algebraic comparison between the mean and median.

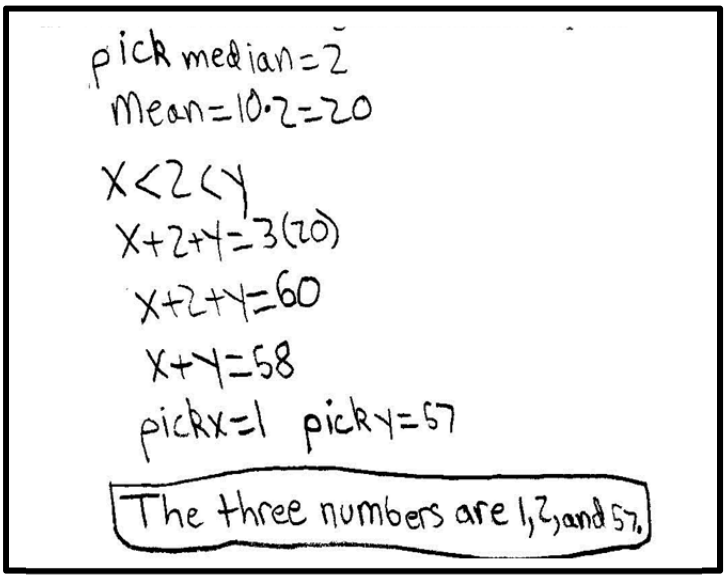

Figure 2. Alex's solution to the Mean and Median Comparison problem.

2. Adding More Components

A traditional exercise involving the Pythagorean Theorem would involve a two-leg journey: *Starting from her home, Jenny drives 60 miles south to visit her grandparents. She then drives 45 miles east to visit her sister Trudy. How far is Trudy's home from Jenny's home?* Adding more components to a problem can make it a fundamentally more sophisticated problem to solve.

> **Multi-Leg Journey:** Starting at her home, Jenny drives 90 miles north to visit her grandparents. She then drives 40 miles west to visit her brother. Next she drives 150 miles south to visit her mother. Finally, she drives 85 miles east to visit her sister Trudy. How far is Trudy's home from Jenny's home?

Students must construct a careful diagram and recognize how the four legs of the journey are equivalent to two legs of a right triangle: Jenny ends up 60 miles south (90 miles north followed by 150 miles south) and 45 miles east (40 miles west followed by 85 miles east). Interpretation and critical thinking are needed to properly apply the Pythagorean Theorem to solve the problem.

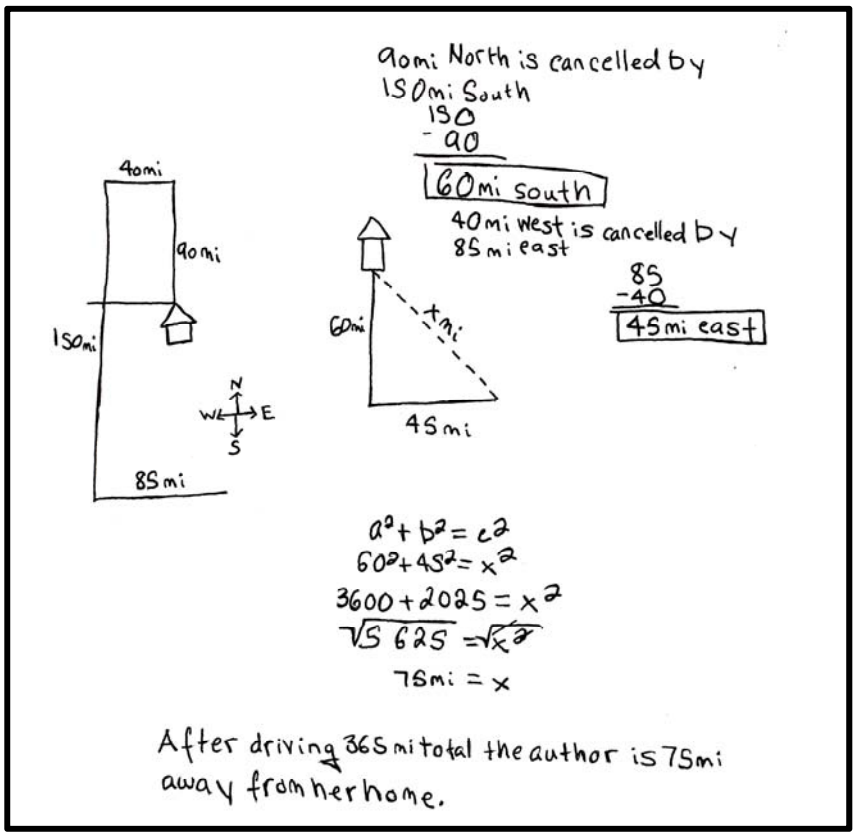

Figure 3. Tim's solution to the Multi-Leg Journey problem.

3. Integrating Content

Too often, mathematical topics are studied in discrete, unrelated chunks. Problems that combine different content areas ask students to integrate their knowledge and combine methods to construct a solution. In the Persian Rug problem, Henry integrates his knowledge of fractal patterns with geometric probability to determine the answer.

Persian Rug Problem. Alex owns a Persian rug with a fractal design, as shown. If a grain of sand randomly falls on the rug, what is the probability that it lands on a shaded region?

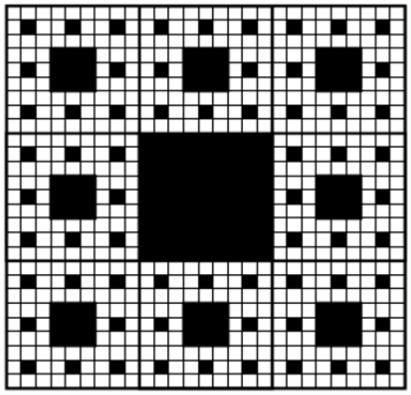

Figure 4. Persian Rug problem.

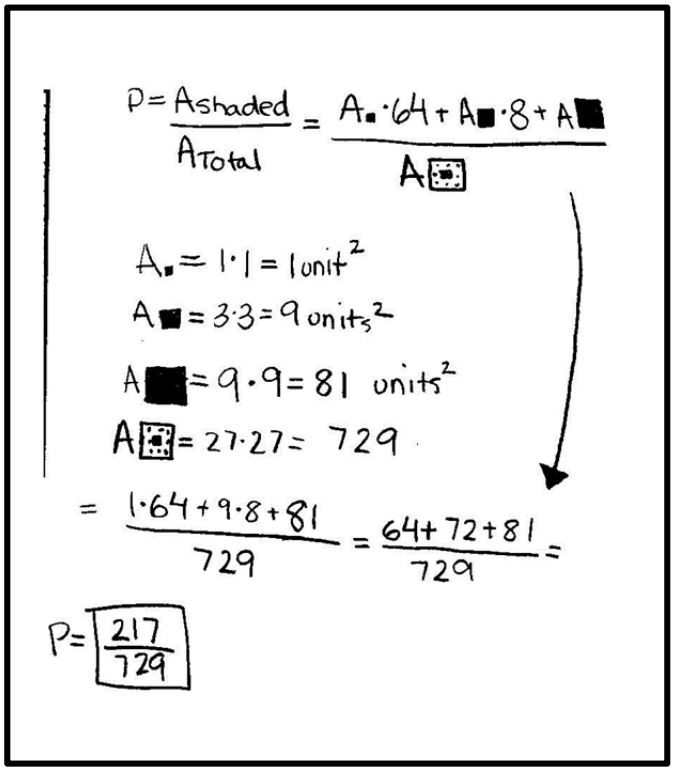

Figure 5. Henry's solution.

Summary

Discussing several problems with student work and also working through some fresh problems will illustrate the goal of creating problem-solving opportunities in a content-rich setting. Some of the points we will stress in this talk will include:

• Don't let content get too far ahead of practice.

• Problem solving opportunities occur when teachers are willing to step back and let students make decisions and create solutions.

• Problem solving does not have to involve only brain teasers, puzzles, contest problems, or period-length investigations: it can involve smaller decision making intimately connected with daily curriculum.

• Authentic mathematical reasoning should allow for some student choice within a curriculum.

For more problems:

Please see:

• Darin Beigie. 2012. Slow-Cooker Problems in a Microwave World. ***Mathematics Teaching in the Middle School*** 18 (September): 76-79.

• Darin Beigie. 2011. No Child Left Unchallenged. ***Mathematics Teaching in the Middle School*** 17 (November): 214-221.

• Darin Beigie. 2011. ***Mathematical Reasoning™ Middle School Supplement.*** The Critical Thinking Company: Seaside, CA, 160 pages.

• Darin Beigie. 2011. The Leap from Patterns to Formulas. ***Mathematics Teaching in the Middle School*** 16 (February): 328-335.

• Darin Beigie. 2008. Integrating Content to Create Problem Solving Opportunities. ***Mathematics Teaching in the Middle School*** 13 (February): 352-360.

• Darin Beigie. 2004. Coordinate Plane Set Detective. ***Mathematics Teaching in the Middle School*** 9 (January): 251-255.

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