**What Comes Before the Standard Algorithm?**

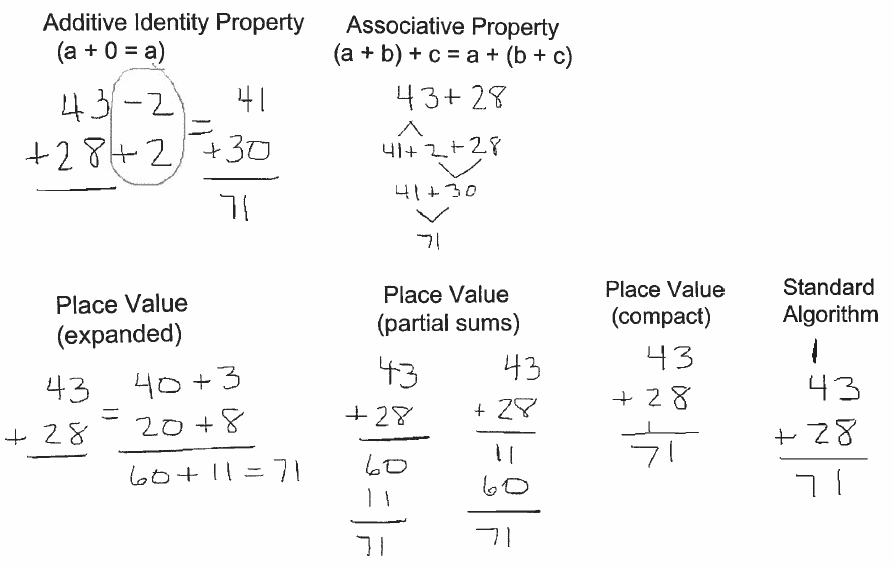
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**Developing Computational Fluency: Multi-Digit Addition and Subtraction**

|  |  |  |
| --- | --- | --- |
|  | CA | CCSS |
| Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums. |  | 1 |
| Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. |  | 1 |
| Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. |  | 2 |
| Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. |  | 3 |
| Fluently add and subtract multi-digit whole numbers using the standard algorithm. | 2,3 | 4 |

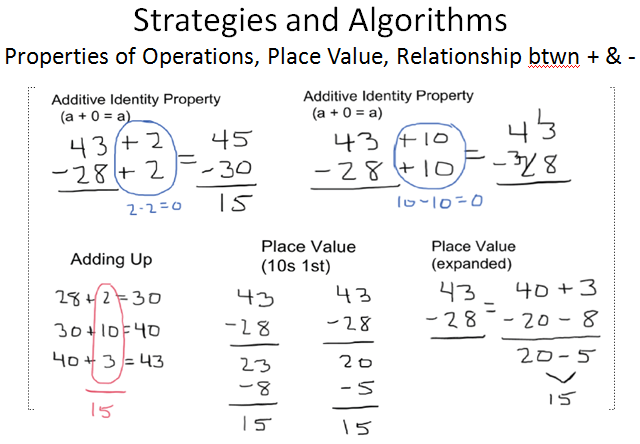
**Multi-Digit Addition with Properties of Operations and Place Value**



**The Properties of Operations (adapted from CCSS-M)**

**(also known as the Field Properties)**

|  |  |  |
| --- | --- | --- |
| **Name of Property** | **Addition** | **Multiplication** |
| Commutative | a + b = b + a | a x b = b x a |
| Associative | (a + b) + c = a + (b + c) | (a x b) x c = a x (b x c) |
| Identity | a + 0 = 0 + a = a | a x 1 = 1 x a = a |
| Inverse | For every a there exists -a so that a + (-a) = (-a) + a = 0 | For every a≠0 there exists 1/a so that ax1/a = 1/a x a = 1 |
| **Property Relating Addition and Multiplication** | | |
| Distributive | a x (b + c) = a x b + a x c | (a + b) x c = a x c + b x c |
|  | a x b + a x c = a x (b + c) | a x c + b x c = (a + b) x c |
|  | (c≠0) | (c≠0) |

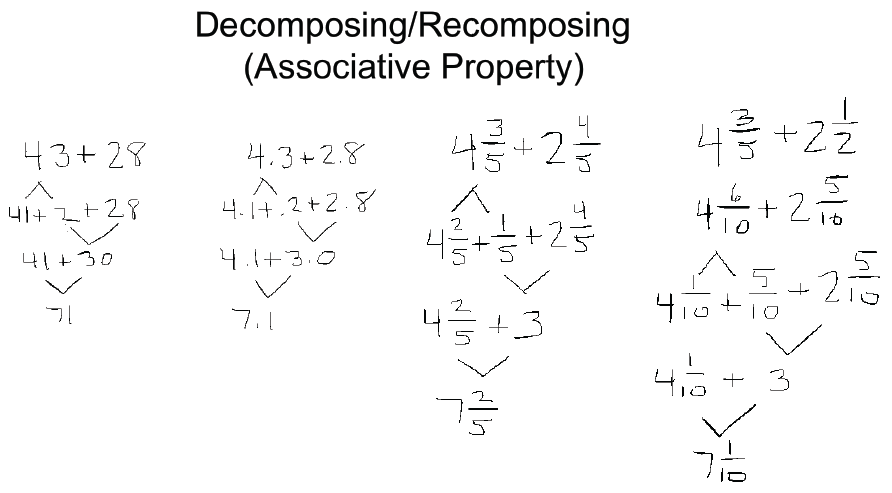
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**Multi-Digit Subtraction with Properties of Operations and Place Value**

**A Framework for Examining Algorithms**

(from Hyman Bass, Teaching Children Mathematics, 2003, as found in Bassareaer, 2005)

* Accuracy (or reliability): the algorithm should always produce a correct answer
* Generality: the algorithm applies to all instances of the problem, or class
* Efficiency (or complexity): this refers to whether the cost (the time, effort, difficulty, or resources) of executing the algorithm is reasonably low compared to the input side of the problem
* Ease of accurate use (vs. proneness to error): the algorithm can be used reasonably easily and does not lead to a high frequency of error in use
* Transparency (vs. opacity): what the steps of the algorithm mean mathematically, and why they advance toward the problem solution, is clearly visible



**Developing Computational Fluency: Multi-Digit Multiplication and Division**

|  |  |  |
| --- | --- | --- |
|  | CA | CCSS |
| Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. |  | 3 |
| Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculations by using equations, rectangular arrays, and/or area models. |  | 4 |
| Fluently multiply multi-digit whole numbers using the standard algorithm. | 4 | 5 |
| Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate an explain the calculation by using equations, rectangular arrays, and/or area models. |  | 5 |
| Fluently divide multi-digit numbers using the standard algorithm. | 5 | 6 |

**Special Strategies**

|  |  |  |
| --- | --- | --- |
| 15 x 18  3 x 5 x 2 x 9  3 x (5 x 2) x 9  3 x 10 x 9  3 x 9 x 10  27 x 10  270 | 17 x 18  17 x 2 x 9  34 x 9  34 x 3 x 3  ? 102 x 3 ?  306 | 17 x 19 |

Don’t work the same way every time.

**General Methods (aka Alternative Algorithms)**

|  |  |  |
| --- | --- | --- |
| 15  X 18  100  50  80  40  270 | 17  X 18  100  70  80  56  306 | 17  X 19  100  70  90  63  323 |

General Methods work the same way every time.

(Note: This Partial Products Alternative Algorithm requires students to pay attention to place values.)

**The U.S. Standard Algorithm**

|  |  |  |
| --- | --- | --- |
| 4  15  x 18  120  15\_  270 | 5  17  X 18  136  17\_  306 | 6  17  x 19  153  17\_  323 |

Standard Algorithms work the same way every time.

(Note: U.S. Standard Algorithms use single-digit computation,

and are often taught without connections to place value concepts.)

