Neuroscience and the Role of Pattern Generalizing in Algebra

Session 565, Mesquite G

November 2, 2013, 8:30 – 10:00 am
Palm Springs Convention Center

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Description:
The brain's primary thinking mode is pattern generalizing; vision is dedicated to mathematical processing. So say Nobel Laureate Gerald Edelman and neuroscientist Steven Pinker. Where does reasoning fit? Sense making? Neuroscientists help here too.

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Investigation **Class**_______________**Name**___________________________________

1. In downloading software for your calculator from the WWW, you notice ______________________________________
your web browser lists the following information about the download:
File size 450K at 230K per second. What do you think these two numbers mean?______________________________

____

2. Would it be faster or slower to download if the browser said 450K at 320K per second? ____________________

3. Would it be faster or slower to download if the browser said 450K at 180K per second? ____________________

4. How long would it take for the file to transfer if it said 450K at 0K per second? ____________________

5. How long would it take to transfer under the conditions of 450K at 450K per second? ____________________

6. If the file size were 225K and the browser transfer window said 225K at 30K per second, ____________________
would it take longer or shorter time to transfer than in Question 1?

7. A Chicago taxi company charges a cab fare of $3 entrance fee and $1.55 per mile. What do ____________________
you think these numbers mean?

____

8. Would your taxi fare rise faster or slower than in Question 7 if the fare structure ____________________
were $3 entrance fee and $1.35 per mile?

9. Would your taxi fare rise faster or slower than in Question 7 if the fare structure ____________________
were $3 entrance fee and $1.60 per mile?

10. How much would your fare be if the fare structure were $3 entrance fee and $0.00 per mile? ____________________

11. Before taxi drivers had computers to calculate fares, they made tables from which they could read the fare. Make a small table for fares from the taxi company in Question 7.

Miles	0.5	0.75	1	1.5	2	2.25	2.5	3	4	5	5.5	6	10
Fare													

Concept Quiz **Class**__________________**Name**____

1. On Route 64 in West Virginia, a road sign indicates that the incline down the mountain is 7%.

   What is the slope of the road?____

2. The graph of $y = \sqrt{2x-3}\sqrt{2x-3}$ is a ray. (half of a line) What is the slope of the ray?____________________________

3. The graph of $y = 3\sqrt{x+2}$ from $x = 10$ to $x = 11$ looks like a line. Explain why it is not a line.

   ____

   ____

4. The roof truss on a house has a horizontal brace of 12 feet and a vertical support of 12 feet.

   What is the slope of the roof?____

5. A popular ski resort advertises a mountain with an 8-mile ski run. If the mountain is 1.5 miles high and the average distance from the bottom center of the mountain to the foot of the mountain is 7.86 miles, what is the slope of the mountain?____

6. Create two different linear functions with *y*-intercepts of 7.____

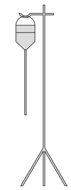
   ____

7. Create two different linear functions with *x*-intercepts of 7.____

   ____

8. Create any linear function with an *x*-intercept of 7 and a *y*-intercept of 7.______________________________

Exploration **Class**________________ **Name**____



1. A 1000-*ml* I.V. drip bag is being administered to a hospital patient at a rate of 2.5 ml per minute. The function that models the amount of I.V. fluid remaining is $Amount = -2.5t + 1000$, where t is time in minutes. When will the I.V. bag have <u>no fluid remaining</u>?

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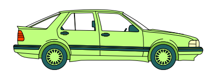
2. A small car (2012 Prius C) with a 9.5-gallon gasoline tank averages 54 miles per gallon when driven. The function that models the amount of gasoline remaining in the gas tank is $G = -\frac{miles}{54} + 9.5$, where *miles* is the number of miles driven. When will the <u>gas tank be empty</u>?



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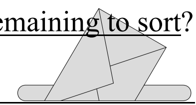
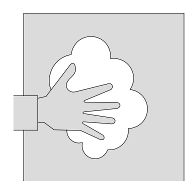
3. A postal worker has 3224 pieces of mail to sort before it can be delivered. He can sort at a rate of 1.2 pieces per second. The function that models the amount of mail remaining to sort is $m = -1.2t + 3224$, where t is time in seconds and m is the mail remaining to be sorted after t seconds have passed. When is there <u>no mail remaining to sort</u>?





4. A window washer in the Dallas - Fort Worth Airport has 873 windows to wash before she can take a break. She can wash windows at a rate of 1 window every 12 seconds. The function that models the number of windows remaining to wash is $w = -\frac{1}{12}t + 873$, where t is time in seconds. Why? When are there <u>no windows remaining to wash</u>?

5. Measurements on a potential customer's house shows a surface area of 1792 square feet, and historic data indicates that you can paint (trim and sides) at a rate of 64 square feet per hour. Thus, the model for the area A remaining to be painted at time t is $A = 1792 - 64t$. When will you be finished with the paint job (<u>nothing remaining to paint</u>)?

6. A tree-watering bag is filled to 1280 ounces (10 gallons) and is set to release 8 ounces per hour. Therefore, the model of the amount of water y_1 remaining in the bag at time x is $y_1 = -8x + 1280$. When is the bag empty (no water remaining)?

Exploration **Class**_______________**Name**___________________________________

Note: A graph of each model (function) may be useful.

1. An I.V. drip contains 500 *ml* of fluid, and the drip rate set by the nurse is at 5 *ml* per minute. If $y = -5x + 500$ models the amount of fluid remaining at time x, when is the bag empty? Also, write the answer as a fraction using the parameters in the model.

   Model: $y = -5x + 500$ Empty when $x =$ ______________________________

2. A tree watering drip bag contains 1200 oz of water, and the drip rate set by the landscaper is 6 oz per hour. If $y = -6x + 1200$ models the amount of fluid remaining at time x, when is this bag empty? Also, write the answer as a fraction using the parameters in the model.

   Model: $y = -6x + 1200$ Empty when $x =$ ______________________________

3. A 50 oz bird bath is being drained at a rate of 4 oz per minute. If $y = -4x + 50$ models the amount of water remaining in the bath at time x, when is the bird bath empty? Also, write the answer as a fraction using the parameters in the model.

   Model: $y = -4x + 50$ Empty when $x =$ ______________________________

4. A 25 oz bird bath is being drained at a rate of 1 oz per minute. If $y = -1x + 25$ models the amount of water remaining in the bath at time x, when is the bird bath empty? Also, write the answer as a fraction using the parameters in the model.

   Model: $y = -1x + 25$ Empty when $x =$ ______________________________

5. If the function $y = -3x + 12$ models an I.V. situation, when is the bag empty? Also, write the answer as a fraction using the parameters in the model.

   Model: $y = -3x + 12$ Empty when $x =$ ______________________________

6. What is the value of the function $y = -3x + 12$, when $x = 4$?______________________________

7. What is the value of the function $y = -2x + 6$, when $x = 3$?______________________________

8. What is the value of the function $y = -5x + 10$, when $x = 2$?______________________________

Note: When $x = 3$ the value of the function $y = -7x + 21$, is 0 (zero). Therefore 3 is called the zero of the function.

9. Find the zero of the function $y = -3x + 12$.______________________________

10. Find the zero of the function $y = -2x + 6$.______________________________

11. Find the zero of the function $y = dx + e$.______________________________

12. An I.V. drip contains 500 *ml* of fluid, and the drip rate set by the nurse is at 5 *ml* per minute. If $y = -5x + 500$ models the amount of fluid remaining at time x, when is the bag empty? What is the zero of the model?

____

Exploration/Concept Quiz **Class**_______________**Name**___________________________________

1. Find the zero(s) of $y = x - 3$ 1. ______________________________________
2. Find the zero(s) of $y = x + 2$ 2. ______________________________________
3. Find the zero(s) of $y = (x - 3)(x + 2)$ 3. ______________________________________
4. Find the zeros(s) of $y = x - 7$ 4. ______________________________________
5. Find the zeros(s) of $y = x - 3$ 5. ______________________________________
6. Find the zeros(s) of $y = (x - 7)(x - 3)$ 6. ______________________________________
7. Find the zero(s) of $y = x + a$ 7. ______________________________________
8. Find the zero(s) of $y = x - b$ 8. ______________________________________
9. Find the zero(s) of $y = (x + a)(x - b)$ 9. ______________________________________
10. Create any function that has a zero of 8 10. ______________________________________
11. Create any function that has a zero of –5 11. ______________________________________
12. Create any function that has zeros of 8 & –5 12. ______________________________________
13. Create any function that has a zero of c 13. ______________________________________
14. Create any function that has zeros of d & c 14. ______________________________________

Investigation **Class**_______________**Name**____________________________________

Polynomial function parameters are the numbers that make-up the coefficients and constants in the polynomial. For example, $f(x) = 5x^2 + 3x - 4$ has parameters of 5, 3, and –4.

1. Find the zeros of $x^2 + x - 12$. ____________________

   Do the function parameters give you an immediate and obvious clue as to what the zeros are?____________________

2. Find the zeros of $x^3 - 13x + 12$. ____________________

   Do the function parameters give you an immediate and obvious clue as to what the zeros are?____________________

3. Find the zeros of $x^4 + 3x^3 - 13x^2 - 27x + 36$. ____________________

   Do the function parameters give you an immediate and obvious clue as to what the zeros are?____________________

When polynomials are written as a product (factored form), the parameters are the coefficients and the constants of the factors. For example, $(dx + e)$ has parameters of d and e.

4. Find the zeros of $(x - 3)(x + 4)$. ____________________

   Do the function parameters give you an immediate and obvious clue as to what the zeros are?____________________

5. Find the zeros of $(x - 3)(x + 4)(x - 1)$. ____________________

   Do the function parameters give you an immediate and obvious clue as to what the zeros are?____________________

6. Find the zeros of $(x - 3)(x + 4)(x - 1)(x + 3)$. ____________________

   Do the function parameters give you an immediate and obvious clue as to what the zeros are?____________________

7. How do you know that $(x - 3)(x + 4)$ can also be written as $x^2 + x - 12$?____

   ____

8. How do you know that $(x - 3)(x + 4)(x - 1)$ can also be written as $x^3 - 13x + 12$?______________________________

   ____

9. If the zeros of a trinomial are 3 and – 2, what is a possible trinomial containing the zeros?________________________

Investigation **Class**_______________**Name**___________________________________

1. If $2x + 5$ is called the function Y_1, is $Y_1(x)$ the function $2x^2 + 5x$? ____________________

2. On your calculator, enter $2x + 5$ in Y_1 and turn it off so that it won't graph. In Y_2 enter $Y_1(x)$ and graph this function. Is it the graph of $2x + 5$ or $2x^2 + 5x$? ____________________

3. If $3x – 1$ is called the function Y_1, is $Y_1(x)$ the function $3x^2 – x$? ____________________

4. On your calculator, enter $3x – 1$ in Y_1 and turn it off so that it won't graph. In Y_2 enter $Y_1(x)$ and graph this function. Is it the graph of $3x – 1$ or $3x^2 – x$? ____________________

5. If $2x^2 – 3$ is called the function Y_1, is $Y_1(x)$ the function $2x^3 – 3x$? ____________________

6. On your calculator, enter $2x^2 – 3$ in Y_1 and turn it off so that it won't graph. In Y_2 enter $Y_1(x)$ and graph this function. Is it the graph of $2x^2 – 3$ or $2x^3 – 3x$? ____________________

7. If $2x^2 – 3$ is called the function Y_1, is $Y_1(4)$ the function $8x^2 – 12$? ____________________

8. Describe, as best you can, the meaning of $f(x) = 7x – 2$. ____________________

9. On your calculator, enter $2x^2 – 3$ in Y_1. On the home screen enter $Y_1(4)$. Does the display show 29 or $8x^2 – 12$? ____________________

10. If $2x + 5$ is called the function Y_1, is $Y_1(3)$ the function $6x + 15$? ____________________

11. On your calculator, enter $2x + 5$ in Y_1. On the home screen enter $Y_1(3)$. Does the display show 11 or $6x + 15$? ____________________

12. If $3x + 4$ is the function Y_1, what is $Y_1(a)$? ____________________

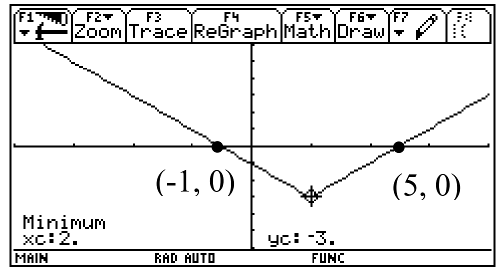
13. If $3x + 4$ is the function Y_1, what is $Y_1(b)$? ____________________

14. If $3x + 4$ is the function Y_1, what is $Y_1(a – b)$? ____________________

15. If $3x + 4$ is the function Y_1, what is $Y_1(a) – Y_1(b)$? ____________________

Investigation **Class**_______________ **Name**_____________________________________

To the right is the graph of the function $y = |x - 2| - 3$ with the zeros and the minimum point marked.



1. For what values of *x* is the function $y = |x - 2| - 3$ zero? ____________________

2. What does this tell you about the roots of the equation $|x - 2| - 3 = 0$?____________________________________

3. If the absolute value function is $y = |x - 2|$, what are the zeros? ____________________

4. What is the root of the equation $|x - 2| = 0$? ____________________

5. If the absolute value function is $y = |x - 2| + 1$, what are the zeros? ____________________

6. What are the roots of the equation $|x - 2| + 1 = 0$? ____________________

7. After studying the original graph above, describe when the graph of the function is below the *x*-axis.______________
____

8. After studying the original graph, describe when $|x - 2| - 3$ is less than zero? ____________________
____

9. If the absolute value function is $y = |x - 2|$, when is it below the *x*-axis? ____________________

10. When is $|x - 2|$ less than zero? ____________________

11. What is the solution to the inequality $|x - 2| < 0$? ____________________

12. If the absolute value function is $y = |x - 2| + 1$, when is it below the *x*-axis? ____________________

13. When is $|x - 2| + 1$ less than zero? ____________________

14. What is the solution to the inequality $|x - 2| + 1 < 0$? ____________________

15. After studying the original graph, describe when the graph of the function is above the *x*-axis.____________________

____

16. After studying the original graph, describe when $|x - 2| - 3$ is greater than zero. ____________________

____

17. If the absolute value function is $y = |x - 2|$, when is it above the *x*-axis? ____________________

18. When is $|x - 2|$ greater than zero? ____________________

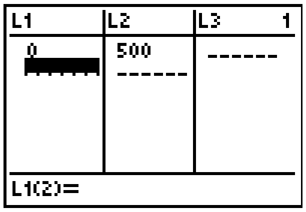
19. If the absolute value function is $y = |x - 2|$, what is the solution to the inequality $|x - 2| > 0$? ____________________

20. If the absolute value function is $y = |x - 2| + 1$, when is it above the *x*-axis? ____________________

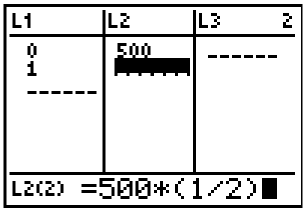
21. When is $|x - 2| + 1$ greater than zero? ____________________

22. What are the roots to the inequality $|x - 2| + 1 > 0$? ____________________

M & M's, Yum! Suppose we have 500 plain and perfect M & M's. We are going to toss them on the table.

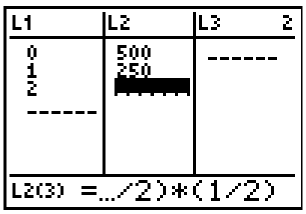


With 0 tosses made, suppose all of the M's are on the top (up). Pick up all the M & M's and toss them on the table



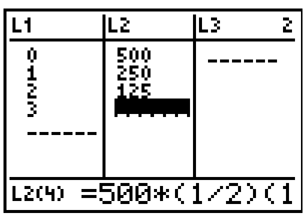
On the first toss, how many of the M & M's do you expect will have the M on the top side?____

How did you get your answer?____________________________________
Eat the ones with the M up and toss the remaining on the table.



On the second toss, how many of the M & M's do you expect will have the M on the top side? ____

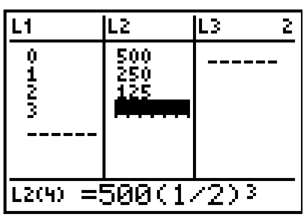
How did you get your answer?____________________________________
Eat the ones with the M up and toss the remaining on the table.



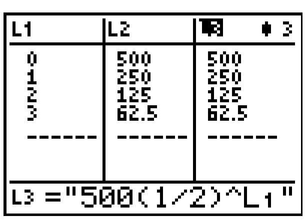
On the third toss, how many of the M & M's do you expect will have the M on the top side?____

How did you get your answer?____________________________________
Eat the ones with the M up and toss the remaining on the table.

What is another way of writing 500∗(1/2)(1/2)(1/2)?_________________

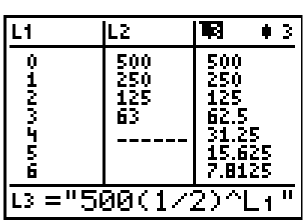


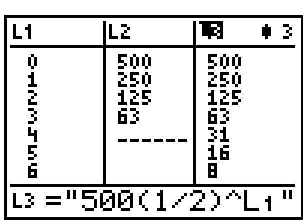
On the L_1th toss, how many of the M & M's do you expect will have the M on the top side?____



How did you get your answer?____________________________________

The natural functioning of the brain will make generalization a relatively straightforward task. It is mathematically a major idea, but easily taught through pattern building.





Please note the issue of the impossibility of 62.5, or 31.125 M's facing up has been solved by doing a MODE of 0 digits after the decimal point.

Concept Quiz **Class**_______________**Name**___________________________________

1. After studying the graphs of functions of the form $f(x) = b^x$, identify what function behavior the parameter b controls.

$g(x)=\left(\frac{1}{15}\right)^x$, $h(x)=\left(\frac{1}{8}\right)^x$, $i(x)=\left(\frac{3}{7}\right)^x$, $j(x)=\left(\frac{5}{6}\right)^x$, $k(x)=1.5^x$, $l(x)=2^x$, $m(x)=3^x$, $n(x)=5^x$, $o(x)=13^x$

____

____

2. After studying the graphs of functions of the form $f(x) = d \times b^x$, identify what function behavior the parameter d controls.

$g(x)=1\cdot 2^x$, $h(x)=2\cdot 2^x$, $i(x)=3\cdot 2^x$, $j(x)=9\cdot 2^x$, $k(x)=-1\cdot 2^x$, $l(x)=-2\cdot 2^x$, $m(x)=-3\cdot 2^x$

____

____

3. After studying the graphs of functions of the form $f(x) = d \times b^x + f$, identify what function behavior the parameter f controls.

$g(x)=-\left(\frac{1}{2}\right)^x+4$, $h(x)=-\left(\frac{1}{2}\right)^x+1$, $i(x)=-\left(\frac{1}{2}\right)^x-1$, $j(x)=-\left(\frac{1}{2}\right)^x-3$, $k(x)=-\left(\frac{1}{2}\right)^x-7$

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4. After studying the graphs of $m(x)=3^x$, $n(x)=3^{x-3}$, $p(x)=3^{x-5}$, $q(x)=3^{x+1}$, $r(x)=3^{x+2}$, $s(x)=3^{x+4}$, identify what function behavior the parameter e controls in the function $f(x) = b^{x+e}$.

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Concept Quiz **Class**________________**Name**____________________________________

Create an exponential function that satisfies the criteria listed below:

1. Is increasing and has a horizontal asymptote at 4.______________________________

2. Is decreasing and has a horizontal asymptote at −3.______________________________

3. Crosses the *y*-axis at 4 and is decreasing.______________________________

4. Has a zero of 2.______________________________

5. Is never positive.______________________________

6. Passes through the point (2, 3).______________________________

7. Lies only in Quadrants III and IV.______________________________

8. Has a horizontal asymptote of $y = n$.______________________________

Exploration **Class**_______________**Name**_____________________________________

1. Make the numeric representations of $y = \frac{x^5}{x^3}$ and $y = x^2$. 1._______________________________

   Are they the same (equivalent)? Explain.

____

____

2. Make the numeric representations of $y = x^2 \cdot x$ and $y = x^3$. 2._______________________________

   Are they the same? Explain.

____

____

3. Make the numeric representations of $y = 3x^{-2}$ and $y = \frac{3}{x^2}$. 3._______________________________

   Are they the same? Explain.

____

____

4. Make the numeric representations of $y = \frac{4x^2 - 2x}{4x^2}$ and $y = 1 - 2x$. 4._______________________________

   Are they the same? Explain.

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Concept Quiz **Class**_______________**Name**___________________________________

1. Find any exponential expression that simplifies through the Product Property of Exponents to 7^{x-1}.

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2. Find any exponential expression that simplifies through the Quotient Property of Exponents to 7^{x-1} (ignore domain differences)

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3. Find any exponential expression that simplifies through the Product Property of Exponents to 3^{3x^2+5x-2} .

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3. Find any exponential expression that simplifies through the Quotient Property of Exponents to 3^{3x^2+5x-2} .

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4. Find any exponential expression that simplifies through the Power Property of Exponents to b^{x^2} .

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6. Find any exponential expression that simplifies through the Power Property of Exponents to $3^{(x-1)(x+2)}$.

Investigation **Class**_______________**Name**___________________________________

1. Explain how you know that $2^3 \cdot 2^x$ is NOT equivalent to 2^{3x}. ____

____

2. Explain how you know that $2^3 \cdot 2^x$ is NOT equivalent to 2^{3^x}. ____

____

3. Explain how you know that $2^3 \cdot 2^x$ is NOT equivalent to 4^{3x}. ____

____

4. Explain how you know that $2^3 \cdot 2^x$ is NOT equivalent to 4^{3+x}. ____

____

5. Is $2^3 \cdot 2^x$ equivalent to 2^{3+x} ? ____________________

6. Explain how you know that $\frac{2^3}{2^{-x}}$ is NOT equivalent to 2^{3-x}. ____

____

7. Explain how you know that $\frac{2^3}{2^{-x}}$ is NOT equivalent to 1^{3-x}. ____

____

8. Explain how you know that $\frac{2^3}{2^{-x}}$ is NOT equivalent to 2^{-3x}. ____

____

9. List one expression that IS equivalent to $\frac{2^3}{2^{-x}}$. ____________________

Exploration **Class**_______________**Name**__________________________________

	$\log d + \log e$	$\log d \cdot e$
1. Find log 2 + log 3 and find the logarithm of the product, log (2×3).	1. 0.7782	0.7782
2. Find log 4 + log 5 and find the logarithm of the product, log (4×5).	2. 1.3010	1.3010
3. Find log 6 + log 7 and find the logarithm of the product, log (6×7).	3. 1.6232	1.6232
4. Find log 8 + log 9 and find the logarithm of the product, log (8×9).	4. 1.8573	1.8573
5. Find log 42 + log 93 and find the logarithm of the product, log (42×93).	5. 3.5917	3.5917

What conjecture can you make about the logarithm $\log g + \log h$? It appears to be the same as $\log (g \times h)$.

	$\log d - \log e$	$\log\left(\frac{d}{e}\right)$
6. Find log 2 – log 3 and find the logarithm of the quotient, $\log\left(\frac{2}{3}\right)$.	6. –0.1761	–0.1761
7. Find log 4 – log 5 and find the logarithm of the quotient, $\log\left(\frac{4}{5}\right)$.	7. –0.0969	–0.0969
8. Find log 6 – log 7 and find the logarithm of the quotient, $\log\left(\frac{6}{7}\right)$.	8. –0.0669	–0.0669
9. Find log 8 – log 9 and find the logarithm of the quotient, $\log\left(\frac{8}{9}\right)$.	9. –0.0512	–0.0512
10. Find log 47 – log 83 and find the logarithm of the quotient, $\log\left(\frac{47}{83}\right)$.	10. –0.2469	–0.2469

What conjecture can you make about the logarithm $\log g - \log h$? It is $\log (g/h)$.

PREPRINT--Modeling for Meaning: A Big Idea

Introduction

What makes mathematical ideas stick? This is a tough question to answer until idea and stick are defined. For the sake of argument, assume that ideas mean abstract algebraic concepts, and for stick, storing algebraic concepts in long-term memory. Unfortunately, a sticking abstract concept does not suggest an understanding of the concept. In this article, the author will propose a process for meeting the challenge of developing student understanding of abstract concepts while storing them in long-term memory.

To meet this challenge, neuroscientist Steven Pinker weighs in on understanding. “The mind couches [understands] abstract concepts in concrete terms” (1997, p. 353). Richard Restak adds to our knowledge of understanding, “We understand something new by relating [connecting] it to something we’ve known or experienced in the past” (2006, p. 164). As for memory, Thompson & Madigan describe one approach to producing sustained memory. “A semantic level of processing, which is directed at the meaning aspects of events, produces substantially better memory for events than a structural or surface level of processing” (2005, p. 33). If this meaning is created by connecting real-world contexts, and previously learned algebraic concepts to the new concept being taught, all that is needed to cause recall of the entire memory is activation of at least two neurons from each concept **(**Seung, 2012). Therefore, capitalizing on core brain function provides a key to the challenge.

If the normal brain requires concreteness in understanding abstract concepts in algebra, it seems that the pedagogy and curriculum should facilitate this common brain function. One path to understanding abstractness through concreteness is modeling meaningful real-world data in teaching symbolic algebra. Modeling data is a standard of the Common Core State Standards for Mathematics (CCSS-M). However, if modeling is treated as just another topic it may suffer a similar fate to any abstract concept/procedure taught unconnected to previous algebra or real-world contexts (Author, 2011b). Confirmation of this notion is found in the CSCC-M: “Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards” (2010, p. 73). In this article, modeling will be used to teach algebraic concepts. Function representation and function behaviors will be the underlying connective theme.

Rationale

Using a conceptual understanding of function representation and function behaviors to teach algebraic concepts presents a perceived problem in course content sequencing. That is, symbolic algebra is often considered a prerequisite to function. However, the CCSS-M argues that integrated or reverse sequencing is not a problem:

> These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time (2010, p. 5).

The CCSS-M further suggests that using function parameters that have a real-world meaning are crucial to a deeper understanding (2010, p. 73). Function parameters in the model below do exhibit a real world meaning, and neuroscientists argue meaning improves understanding and long-term memory (Pinker, 1997; Thompson & Madigan, 2005).

The processes described below use a guided-discovery pattern-building approach to be consistent with the normal brain’s primary mode of thinking – pattern generalizing (Edelman, 2006; Hawkins, 2004; Devlin, 2010; Dawkins, 2009; Eagleman, 2011; Buonomano, 2011; Bor, 2012). When students generalize a perceived pattern, the brain forms a long-term memory and establishes understanding (Hawkins, 2004). Again, it seems that core brain function helps meet the challenge. There is considerable more research evidence from the neurosciences (Author, 2011a); but hopefully, these research references are sufficient to demonstrate the significance of this ubiquitous brain function.

Preparing for the Symbolic Model

In the activity below, students must identify the shape of the data when graphed, and decide if the relationships is increasing or decreasing. The concept of increasing/decreasing is normally established in the brain at a young age, but the I.V. drip and tree bag may need an explanation and a picture before they start the activity. Students need not have a mastery of plotting points because the graphing calculator will graph the data. Upon moving the data to the calculator by running a teacher supplied calculator program, it becomes interactive allowing connections among the numeric, graphic, and English function representations. Connections are associations among distinct neuronal circuits storing and processing the concepts. The pedagogy and/or curricular materials must activate the individual circuits simultaneously to create the associations (McDermott, 2010). Early and frequent use of connections is crucial to meaningful understanding and long-term memory (Author, 2011a; Edelman, 2006; Schacter, 2001; Restak, 2006). An added benefit of using real-world contexts to add meaning is that students can function at a higher cognitive level, and can understand concepts sooner than when no context is used (Greenspan & Shanker, 2004).

Concept Quiz (directions above)

Below is information showing the amount of water *A* remaining in a 40-quart (1280 ounces) tree-watering bag as time *t* passes. The release drip rate is set for 16 ounces per hour.

t	0	1	2	3	4	5	6	12	18	24	30	36	42	48	54	60	66
A	1280	1264	1248	1232	1216	1200	1184	1088	992	896	800	704	608	512	416	320	224

Below is information showing the amount of saline solution *A* remaining in a 1000 ml I.V. drip bag as time *t* passes. The nurse set the drip rate for 4 ml per minute.

t	0	5	10	15	20	25	30	40	50	60	70	80	90	100	110	120	140	160	180	200	220
A	1000	980	960	940	920	900	880	840	800	760	720	680	640	600	560	520	440	360	280	200	120

Below is data showing the amount of water *A* in a pool that contained 7000 gallons at the beginning when a pump started adding water at the rate of 30 gallons per minute.

t	0	1	2	3	4	5	10	20	30	40	50	60	120	180	240	300
A	7000	7030	7060	7090	7120	7150	7300	7600	7900	8200	8500	8800	10600	12400	14200	16000

Below is information describing the amount of saline solution *y1* remaining in a 500 ml I.V. drip bag as time *x* passes. The nurse set the drip rate for 4 ml per minute.

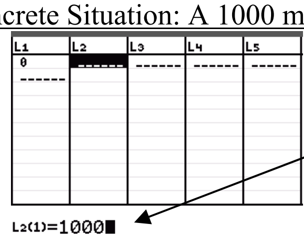
x	0	5	10	15	20	25	30	40	50	60	70	80	90	100	110	120	125
y1	500	480	460	440	420	400	380	340	300	260	220	180	140	100	60	20	0

The data sets above are a subset of a larger classroom set that contains a variety of core function types. The visualizations are used early in the teaching process to capitalize the brain's visual system's relationship to memory and understanding (Pinker, 1997; Schacter, 2001; Lynch & Granger, 2008). This activity primes students for modeling in the next lesson.

Developing Symbolic Form

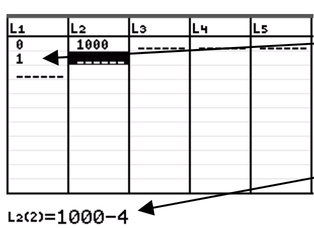
The dialog below between a teacher and students is derived from typical discussions between the author and his students using the method many times, and for a variety of function types.

Concrete Situation: A 1000 ml I.V. drip bag is attached to a patient; the nurse sets the drip rate at 4 ml per minute.



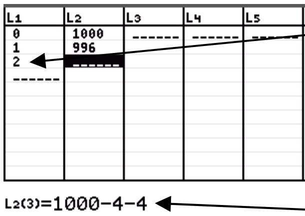
Teacher: If 0 minutes have passed since hanging the I.V. drip, how much fluid remains in the bag?

Student response: Since the bag has 1000 ml to start, it contains 1000 ml at time 0.



Note: Make a table on the board with each input and edit line data.
Teacher: If 1 minute has passed since starting the I.V. drip, how much fluid remains in the bag?
Student response: 996 ml

Teacher: How did you find that?
Student response: 1000 – 4
Notice the numeric representation being created. This will connect the symbolic model to the numeric representations previously analyzed.

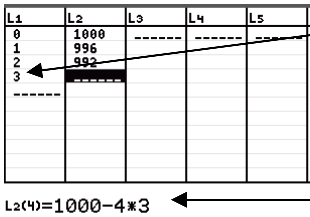


Teacher: If 2 minutes have passed since starting the I.V. drip, how much fluid remains in the bag?
Student response: 992 ml

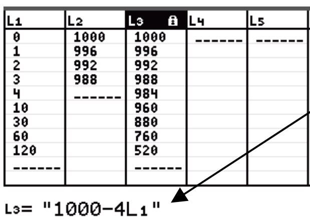
Teacher: How did you find that?
Student response: (996 – 4)

Teacher: Ok and where does the 996 come from?
Student response: 1000 – 4 – 4

The teacher is building a table of the model, but the edit line helps student recognize the desired pattern.

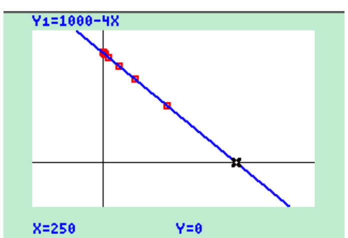


Teacher: If 3 minutes have passed since starting the I.V. drip, how much fluid remains in the bag?
Typically, ***students answer*** 1000 – 4 – 4 – 4 because the normal brain generalizes on the third iteration, but the teacher asks students for another way of writing the expression yielding 1000 − 4×3. At this point, the teacher tries another couple time values to confirm the correct numeric generalization. (Placing 4 before the 3 is significant for generalizing purposes.)



Once the numerical form of the model has been generalized, the next question to ask is how much fluid remains in the bag after L_1 (or *x*) minutes. Having taught this process in 50-75 different classes, the author has never had a class not generalize at this point, no matter what variable for time was chosen.

The emotional tag for the symbols is demonstrated by asking how much fluid remains after 10 minutes, or 60 minutes, or maybe 120 minutes as the symbols calculate related values. Emotional tags are a connection to a common real-world context like the I.V. drip. They enhance connections. Ask students for the meaning of (120, 520). Ask what happens at 250 minutes in this contextual situation.



To make connections among representations, the teacher graphs the data and then the mathematical model with the data. The simultaneity is required to create neural associations (McDermott, 2010). Use trace and scroll along the data and then jump to the model for a variety of data points. Discuss increasing/decreasing to reinforce the connections made from the data sets. Trace to the zero (when the bag is empty). This will prime the students (in a neurological sense) for a lesson on zeros later. Use a decimal window so trace yields integer values for *x* that is desired in a realistic nursing situation.

It is significant to note that function representations have been connected. The symbolic representation of the data set has meaning, students generalized the pattern thereby creating a memory of the algebra, and the increasing/decreasing concepts have been integrated into the neuronal circuitry.

The symbolic model for the I.V. drip may be the student's first experience with linear functions. Students may know nothing about the concepts of "slope" and "*y*-intercept." The choice to use the word rate as opposed to slope is because enduring long-term memory and understanding require meaning and a known connection. Rate is more likely known to a beginning algebra student than is slope.

Continuation: Parameter-Behavior Connections

Below is a student-centered exploration activity designed to further develop the concepts of rate, initial condition (*y*-intercept), and increasing/decreasing. Connections continue to be enhanced and refined. Each time connections are revisited, the neuronal circuitry becomes more mylinated causing a higher likelihood of recall when needed. The exploration requires a graphing tool because students may have limited experience with symbols connected to data sets.

The graphing calculator allows them to explore using TABLE and GRAPH in order to answer the questions. In the activity, students establish a connection between a negative rate of change and a decreasing function. Likewise, for a positive rate and an increasing function. Connections are useful in many ways, like describing function behavior near maximum and minimums.

The guided-discovery activity uses pattern building as a tool to generate understanding and long-term memory of the content. Students need to record the symbolic model, initial condition, the rate, and increasing/decreasing for questions 1 to 5 in a table. The purpose of the table is to help students visualize connections between, for example, the coefficient of *x* with the rate and to increasing or decreasing. From question 6 on, students must generalize the concepts in order to answer each question. Like in the modeling activity, the use of x, y, y_1, etc. seem to have no effect on student's ability to generalize or understand the mathematical concepts.

Recording Table

Question	Model	Initial Amount	Rate of Change	Increasing/Decreasing
1	$y = -3x + 1000$	1000	−3	decreasing
2	⋮	⋮	⋮	⋮

1. The model for the amount of fluid remaining in a 1000 ml I.V. drip set to release 3 ml of fluid per minute is $y = -3x + 1000$. Is the relationship between time *x*, and amount remaining *y*, an increasing or decreasing relationship? At what rate is the amount of fluid in the bag changing?

2. The model for the amount of fluid remaining in a 500 ml I.V. drip set to release 2 ml of fluid per minute is $y = -2x + 500$. Is the relationship between time *x*, and amount remaining *y*, an increasing or decreasing relationship? At what rate is the amount of fluid in the bag changing?

3. Farmer Tom owned 250 acres of land at which point his father-in-law gave him 50 acres a year for tax purposes. The model of the amount *y* of land owned by Tom is $y = 50x + 250$, where *x* is time in years. Is the relationship between time *x*, and amount of land owned *y*, an increasing or decreasing relationship? At what rate is the land Tom owns changing?

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6. Professor Ed's swimming pool water had evaporated down to about 7,000 gallons at which point he returns from vacation and adds water at the rate of 2 gallons per minute. Is the amount of water in the pool increasing or decreasing after the hose is turned on? What is the initial amount in the pool? At what rate is water being added? What is the model of the amount of water $y1$ in the pool if *x* is time in minutes since the start of the pump?

7. Jennifer had a high volume water pump that removed water from the 14,000 gallon pool at the rate of 30 gallons per minute. Is the amount of water in the pool increasing or decreasing after the pump is started? What is the model of the amount of water $y1$ in the pool *x* minutes after the pump is turned on? What is the initial amount in the pool? At what rate is it being removed?

8. If the water level model is $y1 = mx + b$, ($m > 0$) is the water level increasing or decreasing?

9. If the water level model is $y2 = dx + e$, ($d < 0$) is the water level increasing or decreasing?

10. If the water level model is $y3 = dx + e$, ($d < 0$) what is the initial water level?

11. If the model of the water level is $y4 = mx + b$, ($m > 0$) what is the initial water level?

Items in the activity are connected to the previous work, are connected to each other in a sequential order, and will be connected to related concepts like zeros and equations later. The reason formal definitions of slope and *y*-intercept were not included is that learning concepts and procedures needs to be distributed over time while being intermixed with other related concepts and procedures. This process is called interleaving. "Interleaving benefits not only memory for what is studied, but also leads to benefits in the transfer of learned skills" (Bjork, 2013).

Concluding Observations

All normally functioning brains generalize patterns, understand abstract concept through concrete connections and visualizations, and use neural associations to commit mathematical content to long-term memory with recall. This core brain function is not biased toward economic status, race, intelligence, time of day, school location, or teacher experience.

This makes modeling for meaning a good tool for teaching algebra and is a big idea, and at the same time it provides teachers with a pedagogical tool for developing student understanding of algebraic concepts while storing them in long-term memory.

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