The Equation of the Parabola Function I



Since the distance from any point P(x, y) on the parabola to the focus F(0, p) is the same as the distance from P(x, y) to the directrix, y = -p , we can use the distance formula:

$$\sqrt{\left(x-0\right)^{2}+\left(y-p\right)^{2}}=y+p$$

And from this we get $x^{2}+\left(y-p\right)^{2}= (y+p)^{2}$.

Expanding both sides gives $x^{2}+y^{2}-2yp+p^{2}= y^{2}+2yp+ p^{2}$.

Cancelling and regrouping gives $x^{2}= 4yp$ or $\frac{1}{4p}x^{2}= y$. Notice that in the parabola, the x is of second degree and the y is linear.

A similar argument produces the equation of the parabola that opens down.