**Conditional Probability**

**and Independence**

**in the Common Core**

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**From the *Common Core State Standards***

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| **Conditional Probability and the Rules of Probability (S-CP)** | |
| **Understand independence and conditional probability and use them to interpret data** | |
| 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). |
| 2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. |
| 3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. |
| 4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. |
| 5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. |
| **Use the rules of probability to compute probabilities of compound events in a uniform probability model** | |
| 6. Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A,* and interpret the answer in terms of the model. |
| 7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. |
| (+) 8. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. |
| (+) 9. Use permutations and combinations to compute probabilities of compound events and solve problems. |

**Two-Way Tables and the General Addition Rule** (S-CP.1, 4, 7)

**Free Tacos!**

In 2012, fans at Arizona Diamondbacks home games would win 3 free tacos from Taco Bell if the Diamondbacks scored 6 or more runs. In the 2012 season, the Diamondbacks won 41 of their 81 home games and gave away free tacos in 30 of their 81 home games. In 26 of the games, the Diamondbacks won and gave away free tacos. Let W = win and T = free tacos. Choose a Diamondbacks home game at random.

1. Summarize these data in a two-way table.
2. What is the probability that the D-backs win?
3. What is the probability that there are free tacos?
4. What is the probability that the D-backs win and there are free tacos?
5. What is the probability that the D-backs win or there are free tacos?
6. What is the General Addition Rule?

**Conditional probability and independence** (S-CP.3–6)

**More Tacos!**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Win** | **Loss** | **Total** |
| **Tacos!** | 26 | 4 | **30** |
| **No tacos** | 15 | 36 | **51** |
| **Total** | **41** | **40** | **81** |

1. What is the probability that there are free tacos, given that the D-backs won the game?
2. What is the probability that the D-backs win the game, given that there were free tacos?
3. What is a conditional probability? What notation do we use? Is there a formula?
4. When are two events independent? Are the events “D-backs win” and “Free tacos” independent? Justify.

**Gender and Handedness**

|  |  |
| --- | --- |
| **Gender** | **Handed** |
| Female | Right |
| Male | Right |
| Female | Right |
| Male | Right |
| Male | Right |
| Male | Right |
| Female | Right |
| Female | Left |
| Male | Left |
| Female | Right |
| Female | Right |
| Female | Left |
| Male | Right |
| Male | Right |
| Male | Right |
| Female | Right |
| Male | Right |
| Male | Right |
| Female | Right |
| Female | Right |

Using the random sampler at [www.amstat.org/CensusAtSchool](http://www.amstat.org/CensusAtSchool), 20 high school students were selected. The gender and handedness of each student is listed in the data table to the right.

1. Create a two-way table to summarize these data. How is a two-way table different than a data table?

Choose one of these students at random.

1. What is the probability that the student is female and right-handed?
2. What is the probability that the student is female or right-handed?
3. Given that the student is female, what is the probability that she is right-handed?
4. Are “selecting a female” and “selecting a right-hander” independent events? Justify.

**Tree Diagrams, the General Multiplication Rule, and Independence** (S-CP.1, 2, 8)

**Serve It Up!**

Tennis great Roger Federer made 63% of his first serves in the 2011 season. When Federer made his first serve, he won 78% of the points. When Federer missed his first serve and had to serve again, he won only 57% of the points. Suppose we randomly choose a point on which Federer served.

1. Display this chance process with a tree diagram.
2. What is the probability that Federer makes his first serve and wins the point?
3. What is the General Multiplication Rule? What if the two events are independent?
4. What is the probability that Federer wins the point?
5. Given that Federer won the point, what is the probability that he missed his first serve?

**False Positives and Drug Testing**

Many employers require prospective employees to take a drug test. A positive result on this test indicates that the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose that 4% of prospective employees use drugs, the false positive rate is 5%, and the false negative rate is 10%. Select one prospective employee at random.

1. Would it be better to use a tree diagram or a two-way table to summarize this chance process?
2. What percent of prospective employees will test positive?
3. What percent of prospective employees who test positive actually use illegal drugs?