

MATHEMATICAL ASSOCIATION



supporting mathematics in education

73.46 One Parabola or Many?

Author(s): C. R. Holmes

Source: *The Mathematical Gazette*, Vol. 73, No. 466 (Dec., 1989), pp. 323-324

Published by: [The Mathematical Association](#)

Stable URL: <http://www.jstor.org/stable/3619307>

Accessed: 01/07/2013 15:11

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



The Mathematical Association is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*.

<http://www.jstor.org>

which has general solution

$$I(a) = c_1 e^{ab} + c_2 e^{-ab}.$$

Letting $a \rightarrow 0$ shows that

$$I(0) = c_1 + c_2 = \int_0^\infty \frac{1}{x^2 + b^2} dx = \left[\frac{1}{b} \tan^{-1} \frac{x}{b} \right]_0^\infty = \frac{\pi}{2b}.$$

Also, from the alternative form for $I(a)$, as $a \rightarrow \infty$ $I(a) \rightarrow 0$. Hence $c_1 = 0$ and $c_2 = \pi/2b$ giving the value of the integral as

$$I(a) = \int_0^\infty \frac{\cos ax}{x^2 + b^2} dx = \frac{\pi e^{-ab}}{2b}.$$

Based on notes and correspondence from

I: ALAN FEARNEHOUGH, *Portsmouth Sixth Form College, Portsmouth PO3 6PZ*

II: DENNIS ORTON, *Portsmouth Grammar School*

III: FRANK CHORLTON, *Aston University, Birmingham B4 7ET*

73.46 One parabola or many?

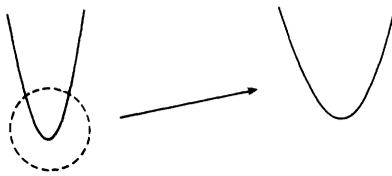
The following makes an interesting problem for sixth forms:

There is only *one* circle shape.
 (All circles can be got from one circle by enlargement.)
 But there are *many* different shaped ellipses.

There is only *one* square shape.
 (All squares can be got from one square by enlargement.)
 But there are *many* different shaped rectangles.

Which category does the parabola come into?
 Are there *many* parabolas, of different shapes?
 Or is there only *one* parabola shape?
 Prove your answer, whichever it is.

It usually leads to heated arguments. Some people observe that the point of a sharp needle under a microscope looks like a blunt needle:

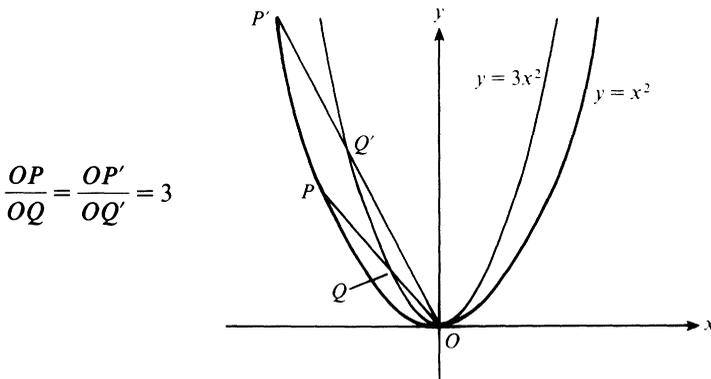


and eventually the conclusion is reached that there is just one parabolic shape.

In general the parabola $y = kx^2$ ($k > 0$) when enlarged by a scale factor of k gives the parabola $y = x^2$:

$$\begin{array}{ccc}
 y = kx^2 & & \\
 \downarrow & \downarrow & \\
 \frac{y}{k} = k\left(\frac{x}{k}\right)^2 & \text{or} & y = x^2
 \end{array}$$

e.g.



C. R. HOLMES

The High School, Boston, Lincolnshire PE21 9PF

73.47 The four points on a parabola and circle

Suppose that a circle intersects a parabola, given parametrically as $(a\lambda^2, 2a\lambda)$, at four points with parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. It is easy to prove (and very familiar to most elderly mathematicians) that $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0$. What may be less familiar is the corresponding result for the natural parameters of the circle.