Expressions, Equations, and Functions in the CCSSM

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Motivation

- In the CCSSM, Algebra and Functions are separate conceptual categories
  - We'll explore why.
- Expressions, Equations, and Functions are three essential objects of study
  - We'll explore how they relate to each other

High School Algebra Standards

There are 4 domains within the Algebra Standards:

- Seeing Structure in **Expressions**
- Arithmetic with Polynomials & Rational **Expressions**
- Creating **Equations**
- Reasoning with **Equations** & Inequalities

What's not in the Algebra Standards?

Functions are not a part of the Algebra Standards

- A separate conceptual category
- A sharp distinction from Algebra as envisioned in the NCTM standards

Why separate Functions from Algebra?

- Algebra is a tool which can be used to study some functions.
- Not all functions have algebraic representations
- The essential objects of algebra are equations and expressions, not functions

High School Functions Standards

- Interpreting **Functions**
- Building **Functions**
- Linear, Quadratic, and Exponential **Models**
- Trigonometric **Functions**

Outline

- An (incomplete) tour of expressions, equations, and functions in the CCSSM with some connections to K-8
- An exploration of how these objects relate to each other
- Throughout, we'll examine some relevant tasks

Quick Tour (Expect some overlap)

Algebra Domains

- Seeing Structure in Expressions
- Arithmetic with Polynomials & Rational Expressions
- Creating Equations
- Reasoning with Equations & Inequalities

Functions Domains

- Interpreting Functions
- Building Functions
- Linear, Quadratic, and Exponential Models
- Trigonometric Functions

Seeing Structure in Expressions

Tour of Algebra & Function Domains

Practice Standard 7

- **Look for and make use of structure**
  - Mathematically proficient students look closely to discern a pattern or structure.
  - They also can step back for an overview and shift perspective.
  - They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects

For the next two tasks, think about how they are connected to PS 7.

Grade 5 SBAC

Can you look for and make use of the structure of these expressions?

1

Several expressions are shown.

Decide if the value of the expression is less than, equal to, or greater than 15.

Drag the expressions to the correct category in the chart.

Less than 15	Equal to 15	Greater than 15

$2 \times \frac{1}{2} \times (5 \times 3)$ $(5 \times 3) \div 5$ $\frac{1}{4} \times (5 \times 3)$

$(5 \times 3) + 6$ $20 - (5 \times 3)$ $(5 \times 3) \times (8 - 7)$

$1 \times (5 \times 3)$ $2 \times (5 \times 3)$

Practice Standard 7

Think about the previous task…

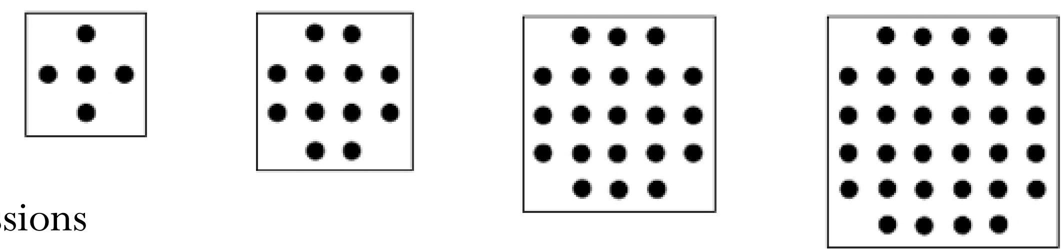
- **Look for and make use of structure**
  - Mathematically proficient students look closely to discern a pattern or structure.
  - They also can step back for an overview and shift perspective.
  - They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects

From the Illustrative Mathematics Project

- illustrativemathematics.org
- Consider the following algebraic expressions: $(n+2)^2 - 4$ and $n^2 + 4n$.

a. Use the figures below to illustrate why the expressions are equivalent

b. Find some algebraic deductions of the same result.



A-SSE: Seeing Structure in Expressions
Cluster: Interpret the structure of expressions.
Standard: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Practice Standard 7

- **Look for and make use of structure**
  - Mathematically proficient students look closely to discern a pattern or structure.
  - They also can step back for an overview and shift perspective.
  - They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects

Creating Equations

Tour of Algebra & Function Domains

SBAC 4th Grade

16

The cost of buying a movie is 4 times the cost of renting a movie. It costs $20 to buy a movie.

A. Choose an equation that can be used to determine the cost of renting a movie, *r*.

B. Drag a number into the box to show the cost of renting a movie.

0
1
2
3
4
5
6
7
8
9

Delete

A.

$20 \times 4 = r$	$20 \div 4 = r$
$4 \times r = 20$	$4 \div r = 20$
$4 \div 20 = r$	$20 \times r = 4$

B. Cost of renting a movie

$ ☐

11th Grade SBAC

- The \$1000 prize for a lottery is to be divided evenly among the winners. Initially there are x winners, but then one more winner comes forward, causing each winner to receive \$50 less. Create an equation that represents the situation and can be used to solve for x, the initial number of winners.
  - An interesting error we recently observed: Students correctly came up with $\frac{1000}{x} - \frac{1000}{x+1} = 50$. To check their answer, they plug in values of x. They conclude that their work must be wrong because they are getting false equations.

Domain: Creating Equations

Cluster Create equations that describe numbers or relationships.

- A-CED.A.1Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
  - A connection to functions.
  - In the SBAC task, the student needs to create an equation in one variable that **could be** used to solve a problem.

Functions

IMP Grade 8 Task

A student has had a collection of baseball cards for several years. Suppose that B, the number of cards in the collection, can be described as a function of t, which is time in years since the collection was started. Explain what each of the following equations would tell us about the number of cards in the collection over time.

A. $B = 200 + 100t$

B. $B = 100 + 200t$

C. $B = 2000 - 100t$

D. $B = 100 - 200t$

IMP High School Functions Task

You put a yam in the oven. After 45 minutes, you take it out. Let $f(t)$ be the temperature of the yam t minutes after you placed it in the oven. In (a)–(d), explain the meaning of the statement in everyday language.

a) $f(0) = 65$
b) $f(5) < f(10)$
c) $f(40) = f(45)$
d) $f(45) > f(60)$

Domain F-IF: Interpreting Functions

Cluster Understand the concept of a function and use function notation.

Standard Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Some Connections

Expressions, Equations, and Functions

Expressions, Equations, and Functions

From the introduction to the Algebra Standards:

"**Expressions can define functions**, and equivalent expressions define the same function. **Asking when two functions have the same value for the same input leads to an equation;** graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling." p. 62

"Expressions can define functions…"

In what sense can an expression define a function?

If the tickets for a concert cost $\$p$ each, the number of people who will attend is $2500 - 80p$. Which of the following best describes the meaning of the 80 in this expression?

A. The price of an individual ticket.

B. The slope of the graph of attendance against ticket price.

C. The price at which no-one will go to the concert.

D. The number of people who will decide not to go if the price is raised by one dollar.

Explain how you chose your answer.

Task from: McCallum, W. G., Connally, E., Hughes-Hallett, D., Cheifetz, P., Davidian, A., Lock, P. F., … Marks, E. J. (2010). *Algebra: Form and Function* (1st ed.). Hoboken, NJ: Wiley.

An Important Standard about Functions

Domain Interpreting Functions

Cluster Understand the concept of a function and use function notation.

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

- Function notation is defined.
- A connection: The graph of a function f is defined as the graph of the equation $y = f(x)$.

Equations & Functions

- **Domain** Reasoning with Equations and Inequalities
- **Cluster** Represent and solve equations and inequalities graphically.
- **A-REI.D.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f and/or g are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Discuss

- **Explain why** the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$.
  - If a point (a, b) is on the graphs of f and g, that means $f(a) = b = g(a)$. In other words, a is a solution to $f(x) = g(x)$.
  - Or…
    - To solve the equation $f(x) = g(x)$ is to answer the question: For what values of x is $f(x) = g(x)$? In other words, for which inputs, do f & g have the same output?
    - If $f(a) = g(a)$, then the point $(a, f(a))$ (aka, $(a, g(a))$) is a point on the graphs of both f and g.

It's always the same question…

$3x = 6$	What are all the values of x which make this true?
$x^2 + 4x + 3 = 0$	What are all the values of x which make this true?
$x^2 + 4x + 3 < 0$	What are all the values of x which make this true?
$3x = y$	What are all the pairs (x, y) that make this true?
$3x - y = 0$	What are all the pairs (x, y) that make this true?
$x^2 + 4x + 3 < y$	What are all the pairs (x, y) that make this true?
$x^2 + y^2 = 1$	What are all the pairs (x, y) that make this true?
$3x + 2y + 3z = 1$	What are all the triples (x, y, z) that make this true?
$kx - y = 0$	What are all the pairs (x, y) that make this true? Or, is it all triples (k, x, y) which make the equation true? By convention, it's typically the former.
$\begin{cases} x + y = 10 \\ x - y = 2 \end{cases}$	What are all the pairs (x, y) which make both equations true?
$3x + 6 = 3(x + 2)$	What are all the values of x which make this true? If an equation in one variable is true for all values of the variable make it true, then the equation is an identity or property.

What's wrong with this task?

Graph the quadratic function to find the solution(s). Identify the solution(s). Explain how you graphed the function.

$$x^2 + 2x = 3$$

Grade 11 SBAC

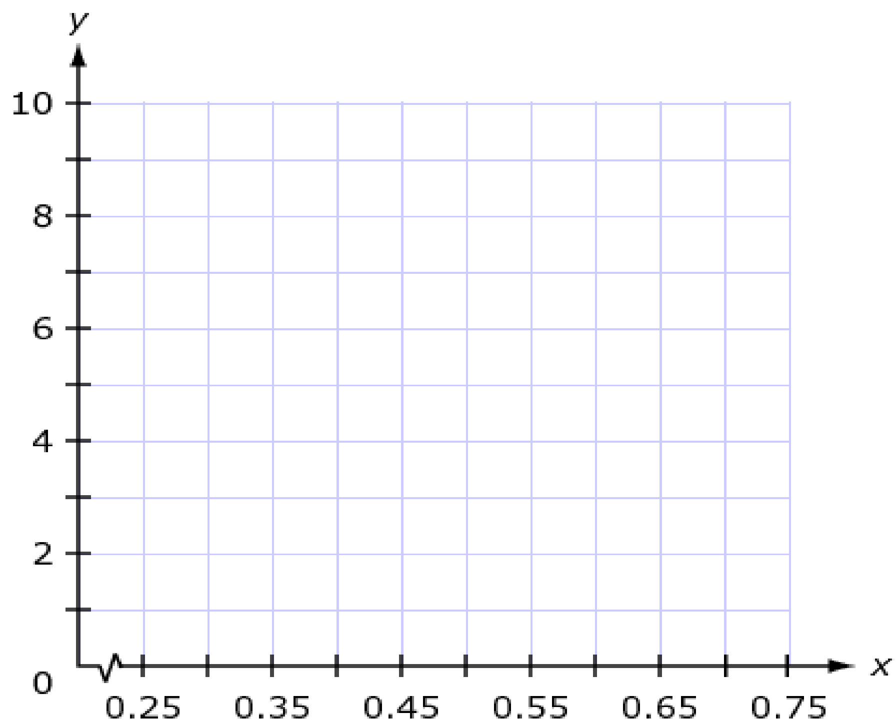
An equation is shown.

$$y = \frac{3}{\sqrt{x}}$$

What does a solution look like for an equation of two variables?

How many solutions are there?

Use the Add Point tool to plot three solutions to this equation on the coordinate grid



Expressions, Equations, Functions

- When does an equation determine a function?
  - Since a function is a relationship between two variables, the equation better have two variables.
    - $x^2 + 2x = 3$ does not have two variables.
  - Do all equations of two variables determine a function?
    - Does $x^2 + y^2 = 1$ determine y as a function of x?
- Can every function be represented by an equation?

- From CCSSM: "Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation" (p. 67)

Equations of One Variable: Connections to Functions

Chazan & Yerushalmy (2003) discuss "different notions of what an equation [of one variable] is":

1. A representation of a set
2. A template for producing sentences about numbers (they are true or false depending on what is chosen for x)
3. A question about the inputs of a function
4. A comparison of two functions of one variable

- Think about the equation $3x + 2 = 7$. How can you think about this equation in each of the four notions listed above?

Chazan, D., & Yerushalmy, M. (2003). On appreciating the cognitive complexity of school algebra: Research on algebra learning and directions of curricular change. *A research companion to principles and standards for school mathematics*, 123–135.

Thank You

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