# Common Core Circles

## Developed by the CMC-S CaCCSSM Committee

# Outcomes

- Participants will
  - Learn how to facilitate student thinking around common core mathematics standards.
  - Learn how to implement such tasks within their classrooms.
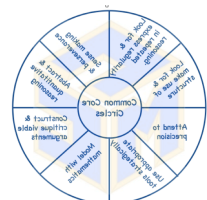  - Receive information on submitting anonymous student work to be part of our study of this and other tasks.

# What is a
# Common Core Math Circle?

- ## Inspired by Math Circles
  - ### Student Math Circles
    - A social context for students to explore math
  - ### Teacher Math Circles
    - A social context for teachers to explore math and its pedagogy.

- ## Common Core Math Circles
  - A social context for teachers to explore Common Core Math Standards and related pedagogy.

http://www.mathteacherscircle.org/news.html

# Team Transport

- Instructions:
  - Solve the task as an adult learner.
  - Once you have solved it, solve it another way.
  - Then solve it in another way...

# Team Transport

- Max is organizing a trip to a football game for 150 students.
  - He can use two types of school buses
  - A small bus costs $80 for the trip and holds up to 8 people.
  - A large bus costs $126 for the trip and holds up to 14 people.
    - 1. (a) If Max orders 6 large buses, how many small buses will he need?
    - (b) How much will the total cost be?
    - 2. Max can organize the journey more cheaply than this!
    - How many buses of each type should Max order, to keep the total cost as low as possible?

# Share Solutions

- Share with others at your tables.

- Facilitator will select representative solutions to share with the whole group.

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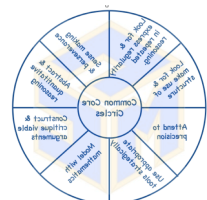
# Make Connections

- **At your table, discuss the following:**
  - What were the key mathematical ideas within the task?
  - What mathematics did you use to solve the problem?
  - How does what you did compare to the solutions presented?

# The Mathematics

- Primary Claim
- Secondary Claim
- Domains/Conceptual Categories
- Standards for Mathematical Practice
- Grade Level Content Standards
- Depth of Knowledge
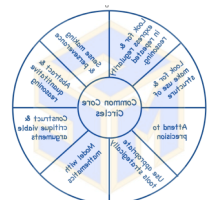
# 4 Claims

- Claim 1: Concepts and Procedures
  - Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency

- Claim 2: Problem Solving
  - Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

- Claim 3: Communicating Reasoning
  - Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- Claim 4: Modeling and Data Analysis
  - Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
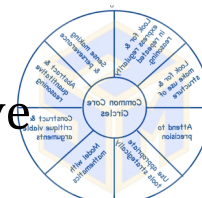
# Domains/Conceptual Categories

| K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | HS |
|---|---|---|---|---|---|---|---|---|---|
| Counting & Cardinality | | | | | | | | | |
| Number & Operations Base Ten | | | | | | Ratios & Proportional Relationships | | | Number & Quantity |
| | | | Number & Operations Fractions | | | The Number System | | | |
| Operations & Algebraic Thinking | | | | | | Expressions & Equations | | | Algebra |
| | | | | | | | | Functions | Functions |
| Geometry | | | | | | | | | Geometry |
| Measurement & Data | | | | | | Statistics & Probability | | | Statistics & Probability |

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# Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

# Depth of Knowledge

- Level 1 – Recall & Reproduction -
    - Recall of information, Performing simple algorithm, Applying a formula.

- Level 2 – Basic Application of Skills/Concepts -
    - Engagement of mental processing beyond a habitual response
    - Make some decisions on how to approach the work.

- Level 3– Strategic Thinking -
    - Reasoning, planning, using evidence, & higher level thinking.

- Level 4 – Extended Thinking -
    - Complex reasoning, planning, developing, & thinking over extended time.
    - Make connections, relate ideas *within* & *among* content areas,
    - Select one approach among many on how to solve a situation.

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# Leaves and Caterpillars

- "*A fourth-grade class needs five leaves each day to feed its 2 caterpillars.  How many leaves would the students need each day for 12 caterpillars?*"

- Use drawings, words, or numbers to show how you got your answer.

- Try to do this problem in as many ways as you can, both correct and incorrect. You may work with a partner.

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# Orchestrating Productive Mathematics Discussions

- Mathematical discussions are a key part of effective mathematics teaching
  - To encourage student construction of mathematical ideas
  - To make student's thinking public so it can be guided in mathematically sound directions
  - To learn mathematical discourse practices

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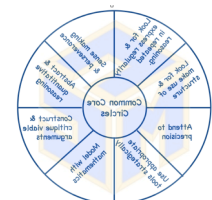
# The Case of David Crane

- Read the handout, "Leaves and Caterpillars: The Case of David Crane"

- As you read the vignette, identify:

  – What aspects of Mr. Crane's instruction would you want him to see as promising?

  – What aspects would you want to help him work on?

Smith, Margaret & Stein, Mary Kay; 5 Practices for Orchestrating Productive Mathematics Discussion; NCTM, 2011.

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# David Crane: What is Promising?

- Students are working on a mathematical task that appears to be both appropriate and worthwhile

- Students are encouraged to provide explanations and use strategies that make sense to them

- Students are working with partners and publicly sharing their solutions and strategies with peers

- Students' ideas appear to be respected

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# David Crane: What Can Be Improved?

- Beyond having students use different strategies, Mr. Crane's goal for the lesson is not clear.

- Mr. Crane observes students as they work, but does not use this time to **assess what students seem to understand** or **identify which aspects of students' work to feature** in the discussion in order to make a mathematical point.

- There is a "show and tell" feel to the presentations.

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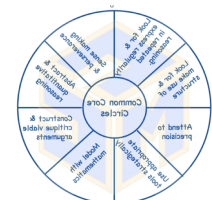
# Conclusion

- The Case of David Crane illustrates the need for guidance in shaping classroom discussions and maximizing their potential to extend students' thinking and connect it to important mathematical ideas.

- What follows is a guide based on five doable instructional practices, for orchestrating and managing productive classroom discussions.
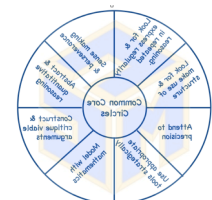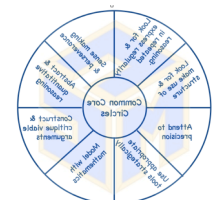
# The Five Practices Model

## What to do in the classroom
## with the task.
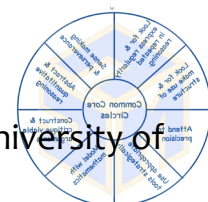
# The Five Practices are:

1. ***Anticipating*** student responses to challenging mathematical tasks;
2. ***Monitoring*** students' work on and engagement with the tasks;
3. ***Selecting*** particular students to present their mathematical work;
4. ***Sequencing*** the student responses that will be displayed in a specific order and
5. ***Connecting*** different students' responses and connecting the responses to key mathematical ideas

# Step 0 (We were kidding about 5)

- Select a task that mirrors the mathematics you want students to learn.

- More about this later!

# 1. Anticipating

likely student responses to mathematical problems

- **Involves considering:**
  - The array of strategies that students might use to approach or solve a challenging mathematical task
  - How to respond to what students produce
  - Which strategies will be most useful in addressing the mathematics to be learned
- **Supported by:**
  - Doing the problem in as many ways as possible
  - Doing so with other teachers
  - Drawing on relevant research when possible
  - Documenting student responses year to year

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# 2. Monitoring

students' actual responses during independent work

- **Involves:**
  - Circulating while students work on the problem and watching and listening
  - Recording interpretations, strategies, and points of confusion
  - Asking probing questions to get students back "on track" or to advance their understanding (no telling!)

- **Supported by:**
  - Anticipating student responses beforehand
  - Using recording tools

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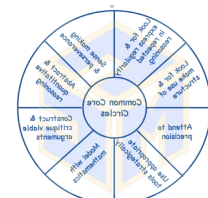
# 3. Selecting

student responses to feature during discussion

- **Involves:**
  - Choosing particular students to present because of the mathematics available in their responses
  - Making sure that over time all students are seen as authors of mathematical ideas and have the opportunity to demonstrate competence
  - Gaining some control over the content of the discussion (no more "who wants to present next")
- **Supported by:**
  - Anticipating and monitoring
  - Planning in advance which types of responses to select
    - Perhaps considering an incorrect solution as it illustrates a typical misconception.
  - Being ready to consider unanticipated solutions.

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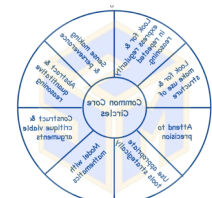
# 4. Sequencing

student responses during the discussion

- **Involves:**
  - Purposefully ordering presentations so as to make the mathematics accessible to all students
  - Building a mathematically coherent story line from prior knowledge to current grade level standards.

- **Supported by:**
  - Anticipating, monitoring, and selecting
  - During anticipation work, considering how possible student responses are mathematically related

# 5. Connecting

student responses during the discussion

- **Involves:**
  - Encouraging students to make mathematical connections between different student responses through questioning
  - Making the key mathematical ideas that are the focus of the lesson salient
  - Considering extensions as they come from the students or the teacher.

- **Supported by:**
  - Anticipating, monitoring, selecting, and sequencing
  - During planning, considering how students might be prompted to recognize mathematical relationships between responses
  - A classroom culture with explicit supports for student discourse.

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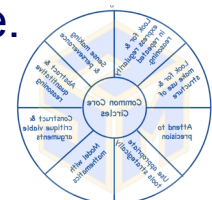
# Why These Five Practices Are Likely to Help:

- **Provides teachers with more control over the learning through**
  - the content that is discussed
  - teaching moves: not everything improvisation
- **Provides teachers with more time to**
  - diagnose students' thinking
  - plan questions and other instructional moves
- **Provides a reliable process for teachers to gradually improve their lessons over time**

# Why These Five Practices Are Likely to Help:

- **Honors students' thinking while guiding it in productive, disciplinary directions**
  - Key is to support students' disciplinary authority while simultaneously holding them accountable to discipline
  - Guidance done mostly 'under the radar' so doesn't impinge on students' growing mathematical authority
  - At same time, students are led to identify problems with their approaches, better understand sophisticated ones, and make mathematical generalizations
  - This fosters students' accountability to the discipline

(Ball, 1993; Engle & Conant, 2002)

# The Goal of Common Core Circles

- For CMC-S:
  - To ensure that all CMC-S affiliates, through Common Core Math Circle modules developed by the Committee, can provide ample opportunities for teachers in Southern California to be prepared for the implementation of the Common Core.
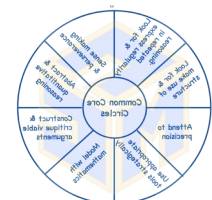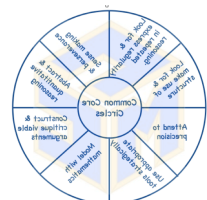
# The Goal of Common Core Circles

- To increase _**teachers'**_
  - mathematical knowledge
  - use of interactive, student-centered problem solving
  - belief in their own mathematical ability.
  - belief in their students' mathematical ability.
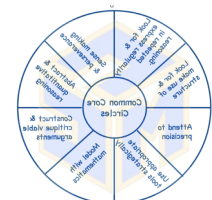  - understanding of Common Core Standards

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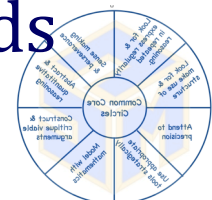
# Student Work

- Please use this task with your students and e-mail us scanned copies of student work that meet the following characteristics:
    - Anonymous (blot out the names)
    - Representative of your students
    - Readable

# Web Site and EMail

- cmc-south.org/caccssm.html

- [commoncorecircles@gmail.com](mailto:commoncorecircles@gmail.com)

# Resources Related to the Five Practices

- Smith, M.S., Hughes, E.K., & Engle, R.A., & Stein, M.K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School, 14* (9), 549-556.

- Stein, M.K., Engle, R.A., Smith, M.S., & Hughes, E.K. (2008).Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning, 10,* 313-340.

- Smith, M.S., & Stein, M.K. (in press). *Orchestrating Mathematical Discussions*. National Council of Teachers of Mathematics.

- Smith, Margaret & Stein, Mary Kay; 5 Practices for Orchestrating Productive Mathematics Discussion; NCTM, 2011.

- NCTM Seminar: Effective Mathematics Instruction: The Role of Mathematical Tasks; Peg Smith University of Pittsburg

- Annenberg Learner: www.learner.org/workshops/missinglink/pdf/tools3.pdf

- Source: Adapted from *Connecticut Common Core of Learning, Mathematics Assessment Project.* Sponsored by a grant from the National Science Foundation.