

The Mathematics of Angry Birds

CMC South 2013

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The Main Objectives

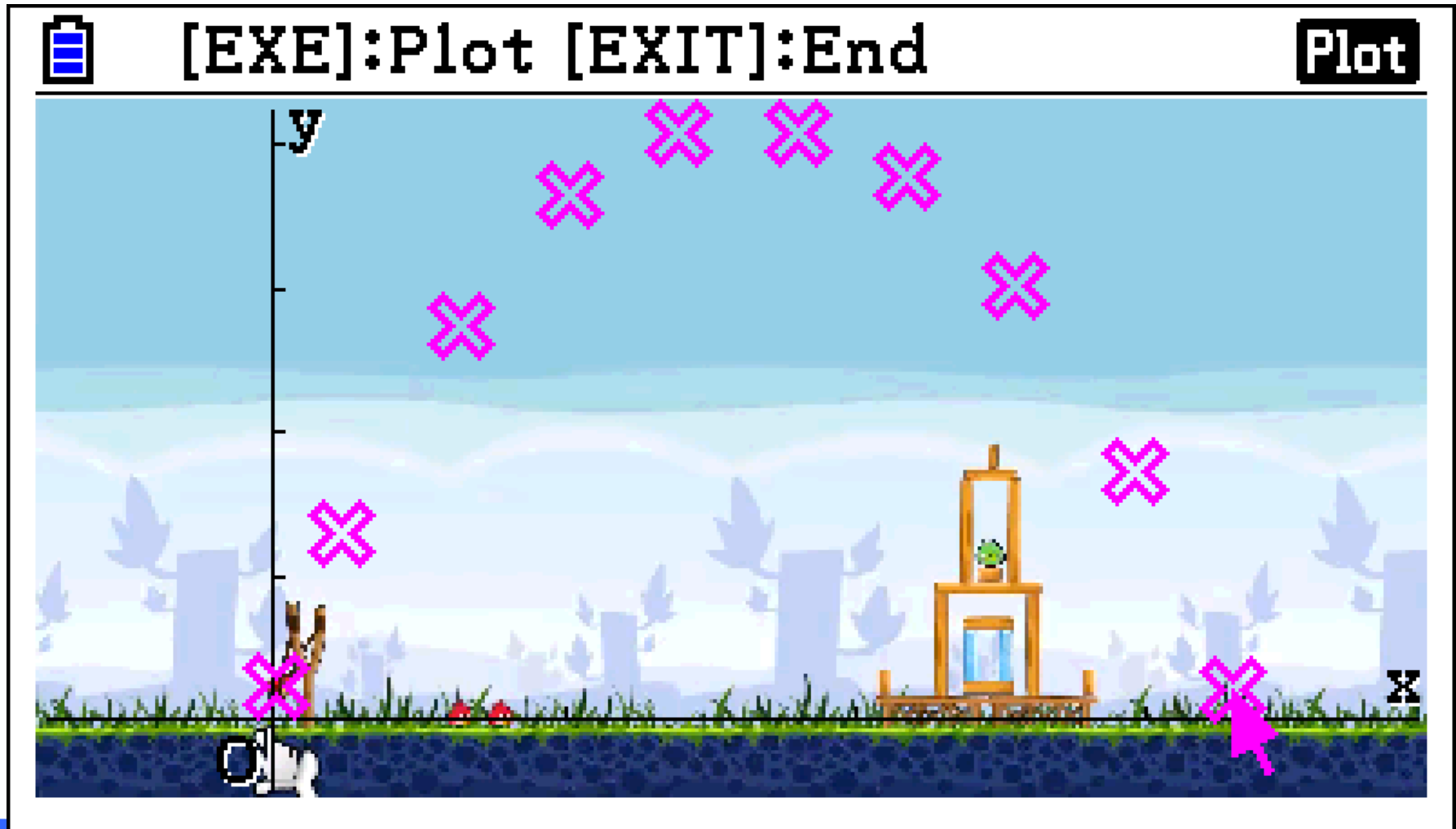
- Use data from the flight of an angry bird to develop models for the motion
- Explore the major variables of angle and initial velocity
- Explore the parametric relation

$$x = (v_0 \cos \theta)t + x_0$$

$$y = -0.5gt^2 + (v_0 \sin \theta)t + y_0$$



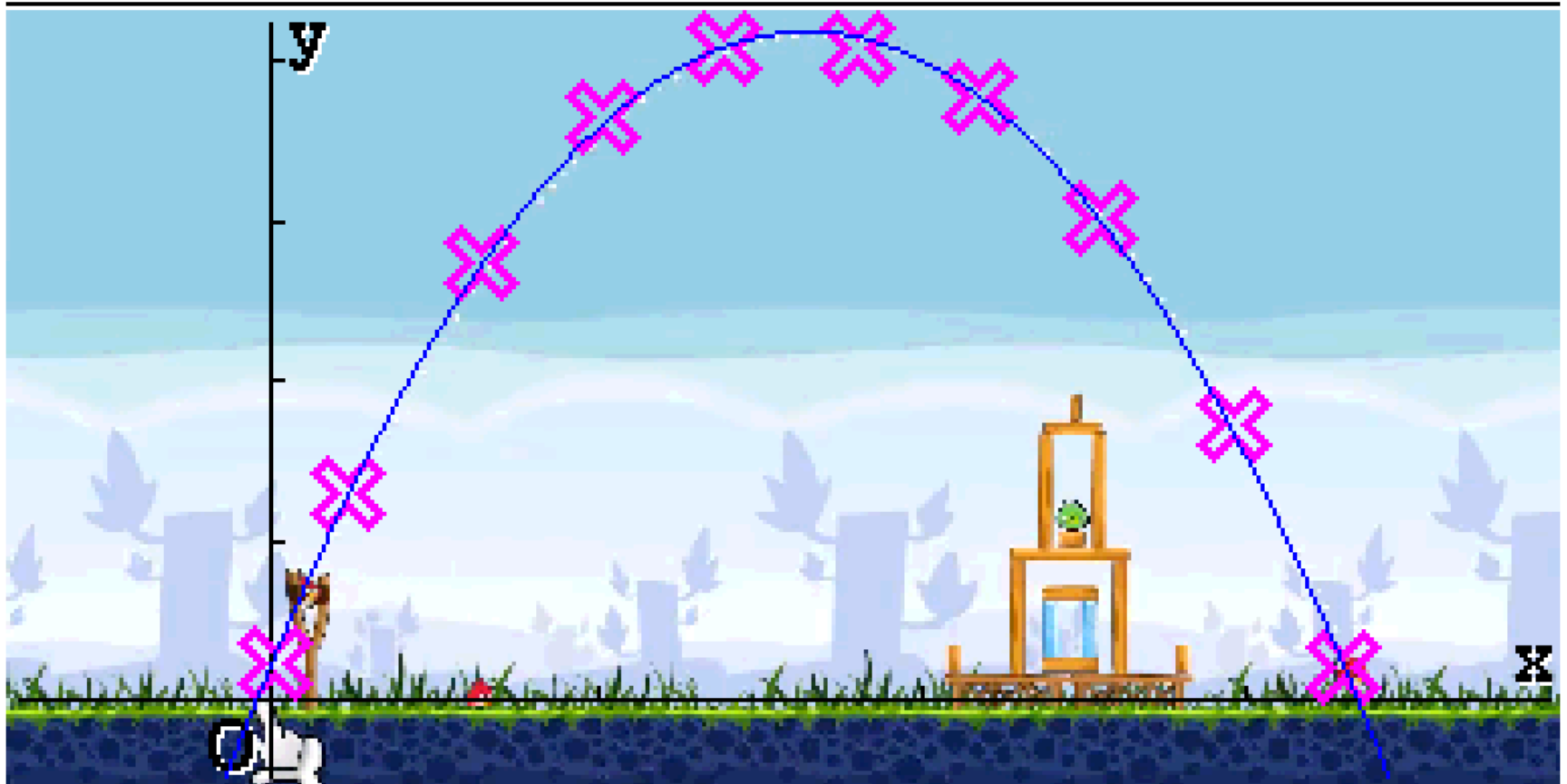
A Demonstration of Data Collection



Create a Model by Regression



Press [OPTN]



More About These Coefficients Later!



QuadReg

$$a = -0.0145123$$

$$b = 2.39544602$$

$$c = 5.98181537$$

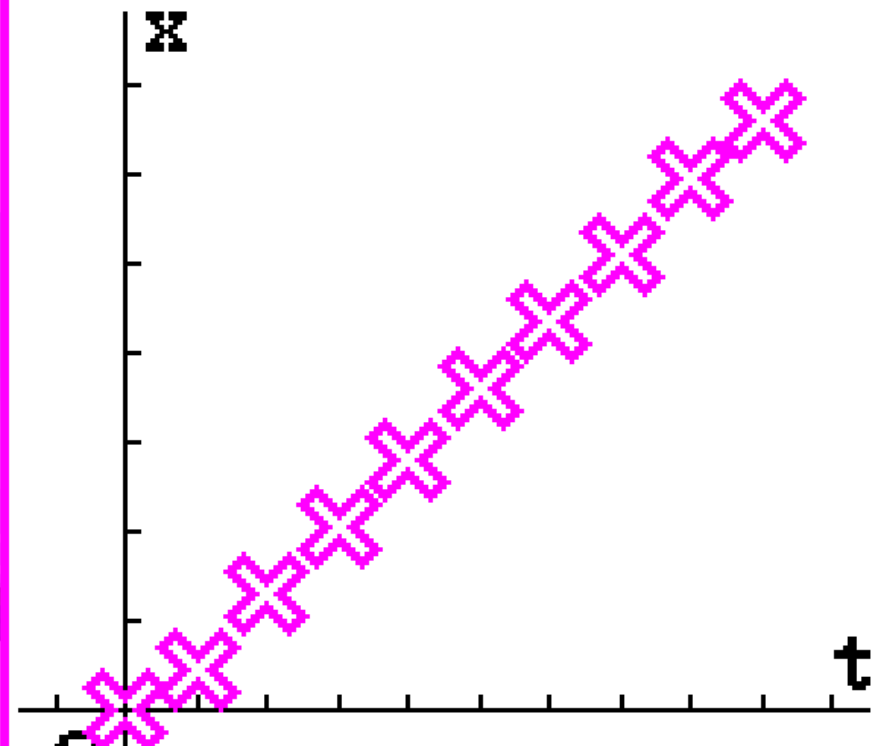
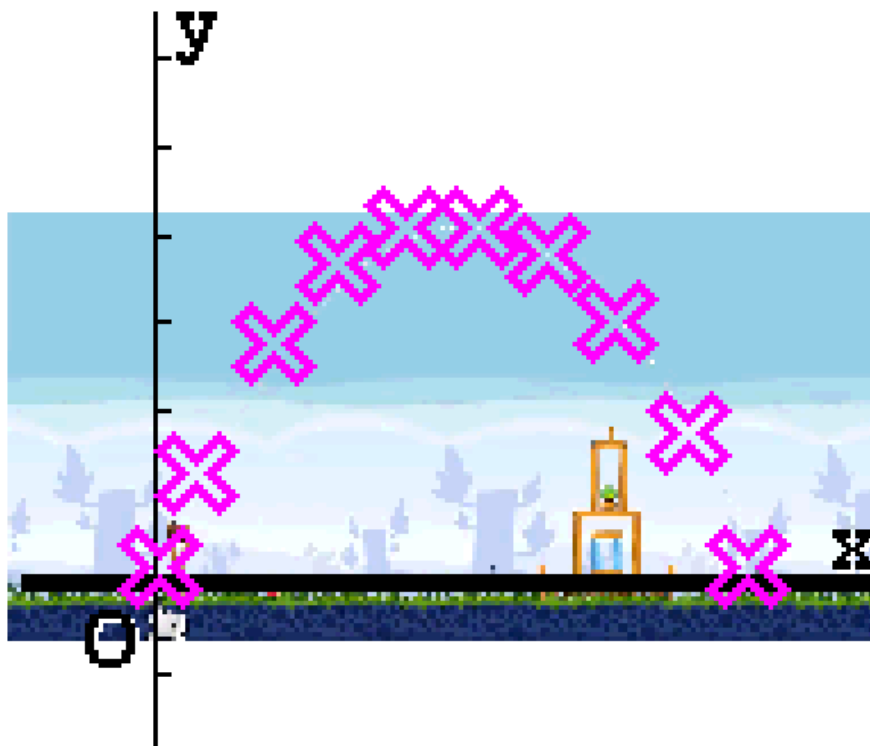
$$r^2 = 0.99938941$$

$$\text{MSe} = 1.1348274$$

$$y = ax^2 + bx + c$$

COPY DRAW

Let's Explore Horizontal Position vs. Time



Switch

List

REG

P-LINK

Interpret Both Coefficients of the Model



Linear Reg (a+bx)

$$a = -4.4002886$$

$$b = 18.866378$$

$$r = 0.99933381$$

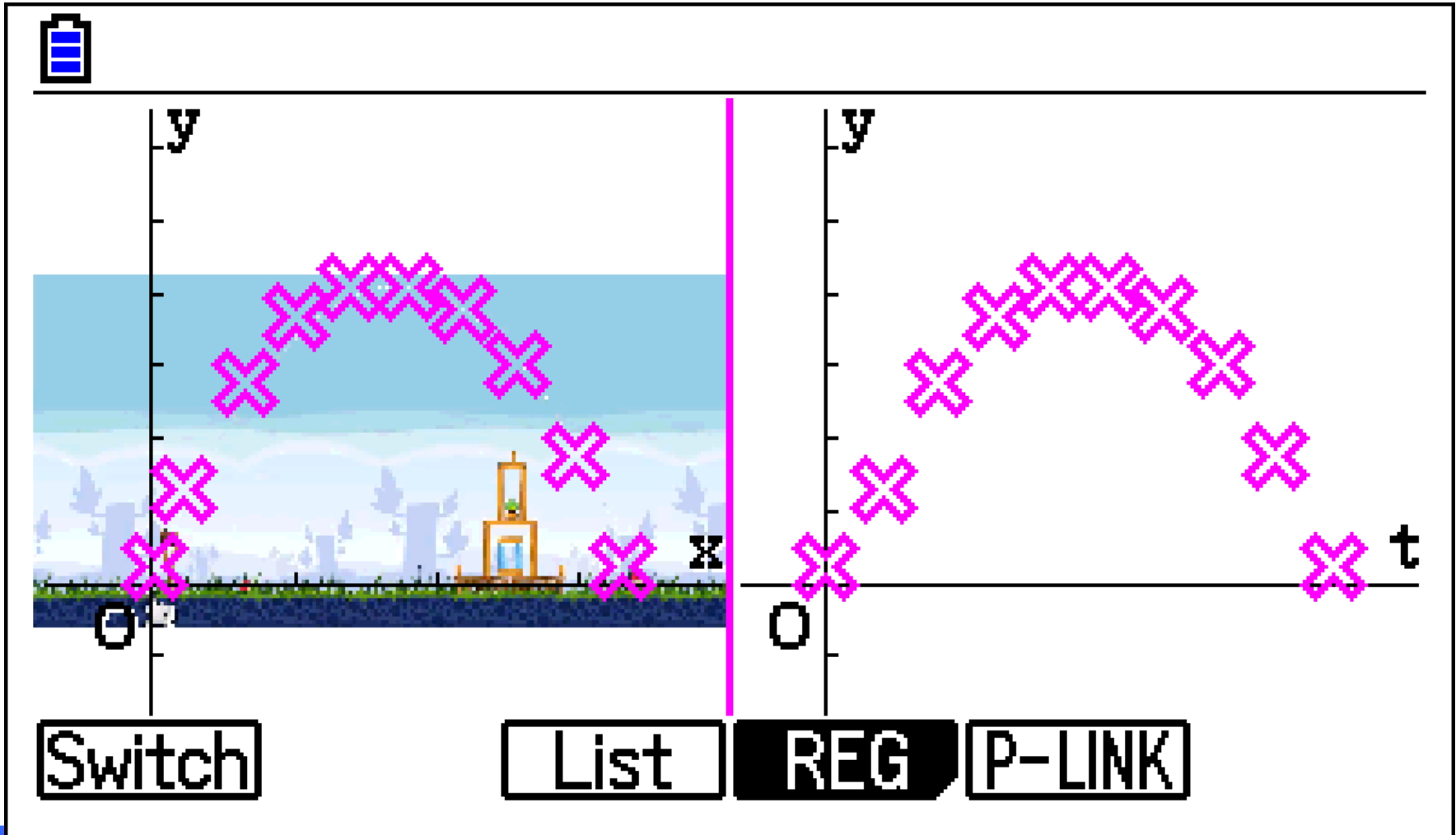
$$r^2 = 0.99866807$$

$$MSe = 4.89551753$$

$$x = a + bt$$

COPY **DRAW**

Let's Explore Vertical Position vs. Time



Interpret the Coefficients of this Model



QuadReg

$$a = -4.9358911$$

$$b = 45.1151026$$

$$c = -0.2461388$$

$$r^2 = 0.99177329$$

$$MSe = 15.2901893$$

$$y = at^2 + bt + c$$

COPY

DRAW

Let's Explore the Angle and Initial Velocity

$$v_0 \sin(t) = 45.115$$

$$v_0 \cos(t) = 18.866$$

- There are several ways to compute the values

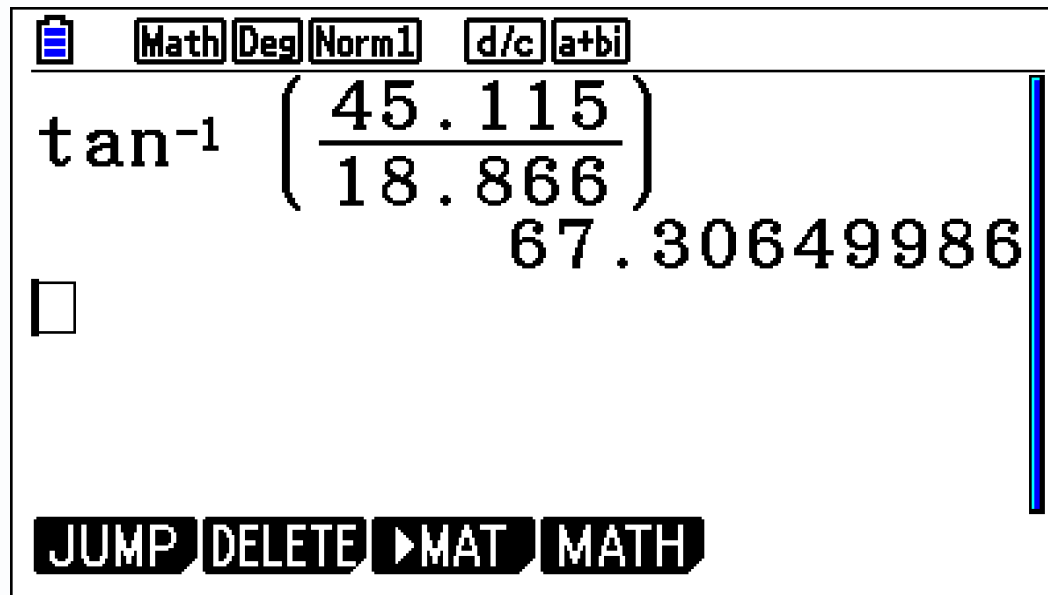


Method 1 - Division

$$v_0 \sin(t) = 45.115$$

$$v_0 \cos(t) = 18.866$$

$$\tan(t) = 45.115/18.866$$



Calculator display showing the calculation of the inverse tangent of the ratio 45.115/18.866. The mode is set to Math, Deg, Norm1, d/c, and a+bi. The result is 67.30649986.

Math Deg Norm1 d/c a+bi

$\tan^{-1} \left(\frac{45.115}{18.866} \right)$

67.30649986

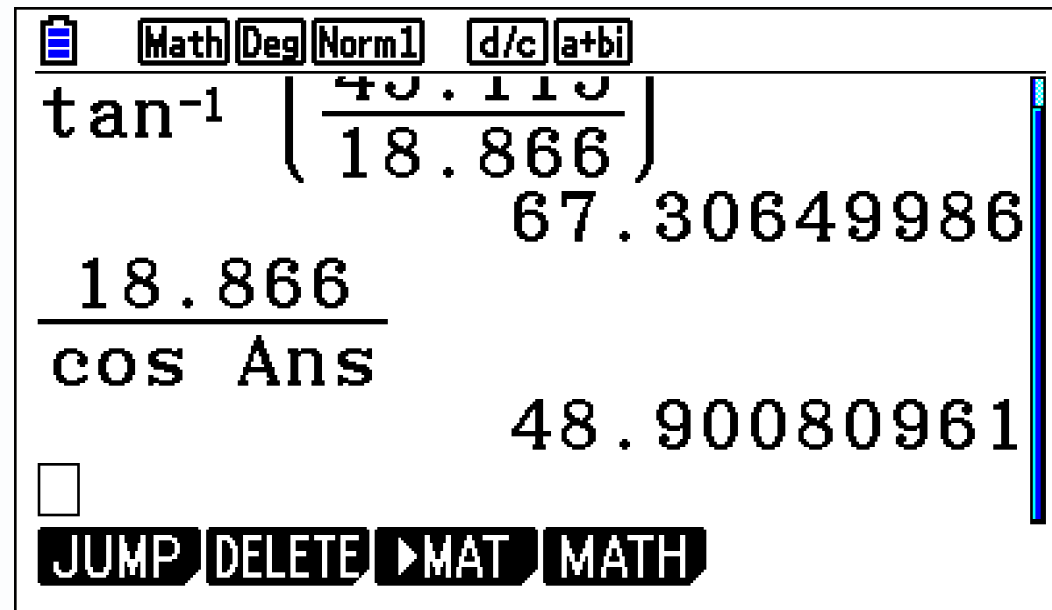
JUMP DELETE ▶ MAT MATH



Method 1 - Substitution

$$v_0 \cos(t) = 18.866$$

$$v_0 = 18.866 / \cos(t)$$



The screenshot shows a Casio calculator interface with the following elements:

- Mode buttons: **Math**, **Deg**, **Norm1**, **d/c**, **a+bi**
- Function key: **tan⁻¹**
- Input: $\left(\frac{48.90080961}{18.866} \right)$
- Result: **67.30649986**
- Input: **18.866**
- Function key: **cos**
- Label: **Ans**
- Result: **48.90080961**
- Bottom navigation buttons: **JUMP**, **DELETE**, **MAT**, **MATH**

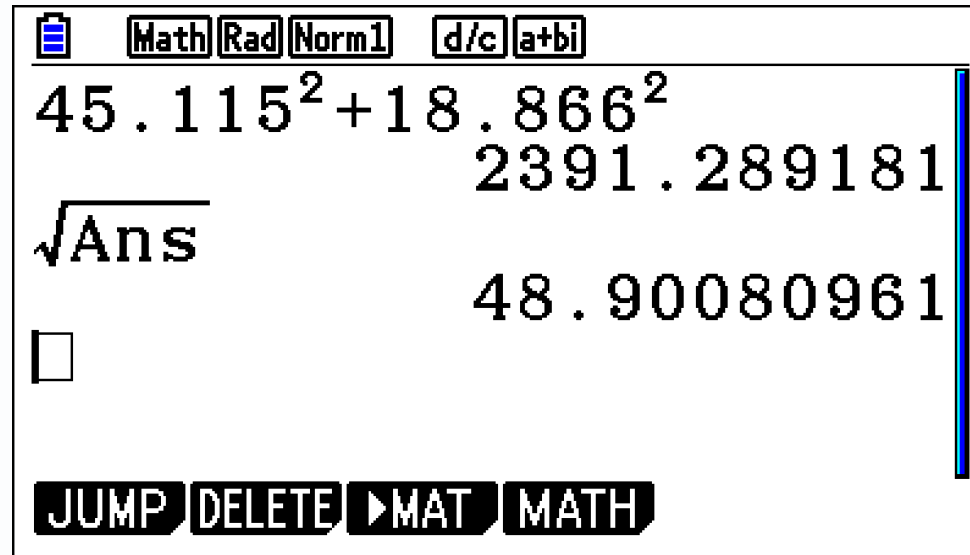


Method 2 – Trig Identity

$$[v_0 \sin(t)]^2 + [v_0 \cos(t)]^2 = v_0^2$$

$$v_0 \sin(t) = 45.115$$

$$v_0 \cos(t) = 18.866$$



The image shows a Casio calculator screen with the following display:

- Mode: **Math** | **Rad** | **Norm1** | **d/c** | **a+bi**
- Input: $45.115^2 + 18.866^2$
- Result: 2391.289181
- Operation: $\sqrt{\text{Ans}}$
- Result: 48.90080961
- Bottom buttons: **JUMP** | **DELETE** | **▶MAT** | **MATH**



Checking the Parametric Model

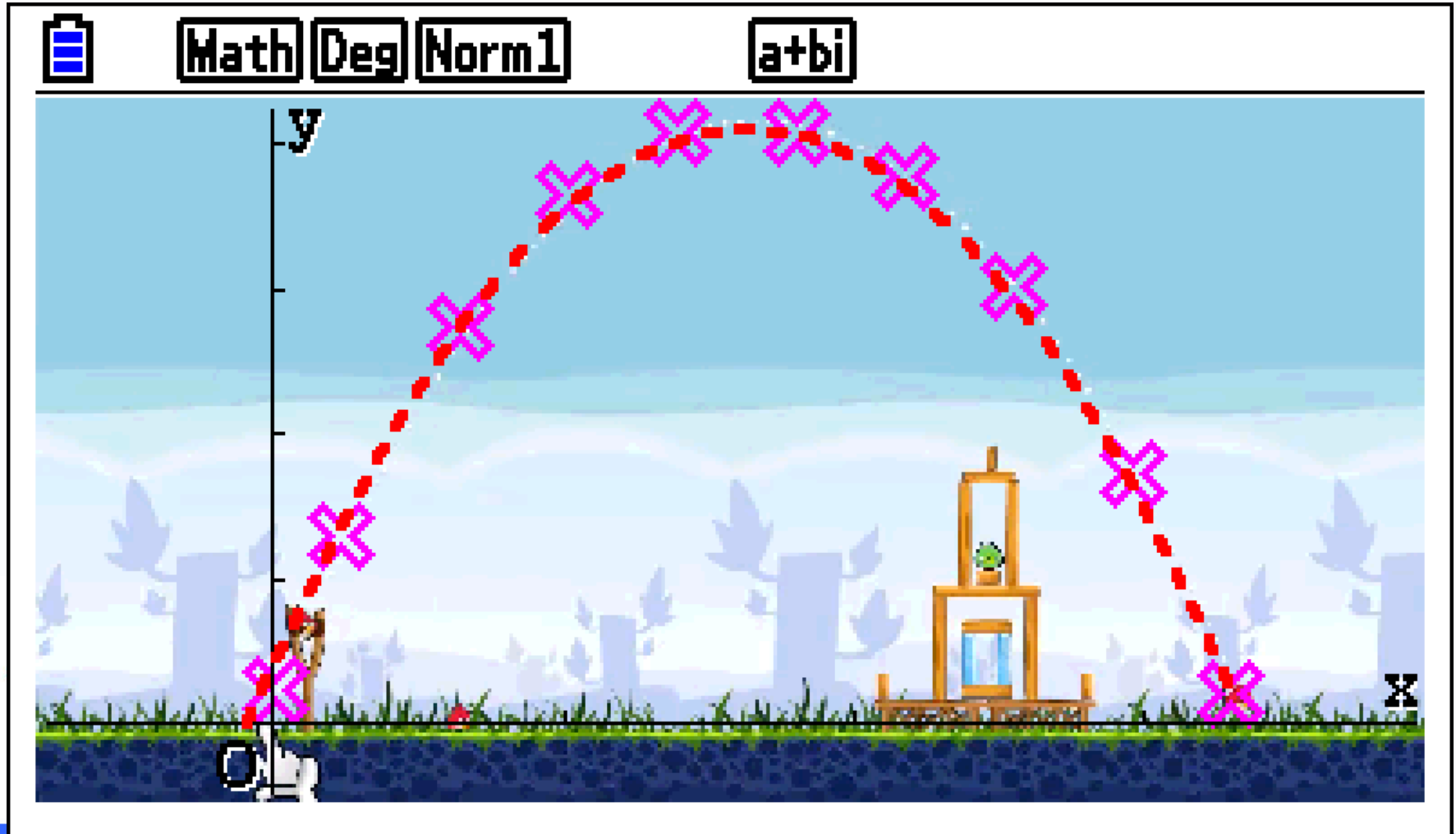
$$X = 48.90 \cos(67.31)T - 4.40$$

$$Y = -4.94T^2 + 48.90 \sin(67.31)T - 0.25$$

```
Math Deg Norm1 a+bi
Graph Func : Param
Xt1 (48.9cos 67. [----]
Yt1 -4.94T^2 + (48.
Xt2: [—]
Yt2:
Xt3: [—]
Yt3:
[SELECT] [DELETE] [TYPE] [TOOL] [MODIFY] [DRAW]
```



Checking the Parametric Model



Revisiting the (x, y) Model

$$x = (v_0 \cos \theta)t$$

$$\frac{x}{v_0 \cos \theta} = t$$

$$y = \frac{-1}{2}gt^2 + (v_0 \sin \theta)t + y_0$$

$$y = \frac{-1}{2}g \left(\frac{x}{v_0 \cos \theta} \right)^2 + (v_0 \sin \theta) \left(\frac{x}{v_0 \cos \theta} \right) + y_0$$

$$y = \frac{-g}{2(v_0 \cos \theta)^2} x^2 + (\tan \theta)x + y_0$$



Revisiting the (x, y) Model

Math Deg Norm1 d/c a+bi

$$\frac{-4.9}{(48.9 \cos 67.3)^2 - 0.01375989768}$$

tan 67.3 2.39057695

DEL-LINE DEL-ALL

QuadReg

a = -0.0145123
b = 2.39544602
c = 5.98181537
 $r^2 = 0.99938941$
MSe = 1.1348274

$$y = ax^2 + bx + c$$

COPY DRAW



Explorations

Use the graphs of (t,x) , (t,y) , and (x,y) to compute the results:

Q1 What is the bird's position at time $t=2.5$ seconds?

Q2 How long is the bird in flight?

Q3 What is the time when the bird is at maximum height?

Q4 What is the maximum height?

Q5 At what time(s) is the bird at height 60 meters?

Q6 How far did the bird fly horizontally?

Q7 What is the height when the horizontal position is 150 meters?

Q8 What is the horizontal position when the height is 60 meters?