Strike a Pose: Modeling in Algebra

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Outline of Workshop

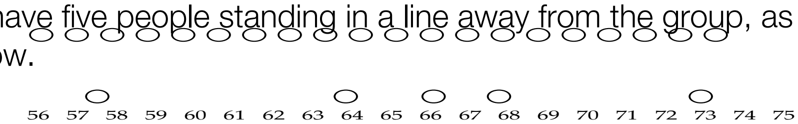
1. Runway versus Mathematical Modeling
   a. Similarities
   b. Differences
2. Common Core State Standards
   a. Standards of Mathematical Practice #4
   b. High School – Algebra
3. Modeling Experiences
   a. Homecoming Dilemma
   b. How Old Is That Artifact?
   c. Use Appropriate Tools Strategically
   d. World's Largest Wave

A complete handout with investigations and materials used in the workshop are available through the CMC South website or at the following website (or scan QR code): http://north-morris.net.temp.guardedhost.com/jennifer-north-morris.html

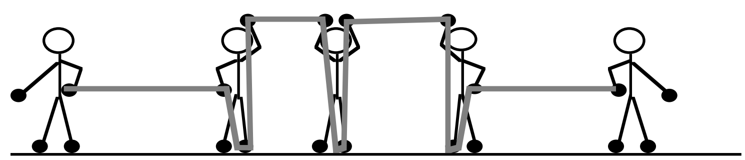
Human Box Plot

Have the students arrange themselves in a line according to their hand spans. As they are arranging themselves, place prepared cards, or mark the floor at equal intervals, with the numbers 15 to 30 to represent the hand spans of the students in centimeters. You may need to adjust the minimum or maximum to fit your class situation. These numbers should be in a line parallel to the student line a few feet away.

- Find the median person and have him or her position themselves in front of the number that is his or her hand span. (If the median is between two people, choose one of them—perhaps by a coin toss—to stand at the mean of their two spans.)
- Find the median student of each half of the split group and have each of these students stand in front of his or her span. (See the note above if the median is between two students.)
- Have the first and last person in the line stand in front of their hand spans.
- You should have five people standing in a line away from the group, as in the diagram below.



- Starting at one end, pass the clothesline from person to person among the five students who are standing on the numbers, having them hold the line to resemble a box plot, as shown in the diagram below.



The five students serve as representatives of the entire group. Guide a discussion on how this graph was formed and the various components of the graph—the median, first and third quartiles, maximum and minimum, and outliers, if any.

Extension: Create two box plots on the same scale. One box plot representing the males in the class and the other representing females. Discuss the contrast (if any) that is pictured.

Human Histogram

C Place the hand span numbers along a line on the floor. Decide as a class what interval width (or grouping size) you will use for the human histogram. For example, you might lay down the numbers 10, 13, 16, . . . , 31 if you decide on an interval width of 3 centimeters. Have each student stand in a line, perpendicular to the number line, in the proper interval. A student whose hand span is 12 centimeters will stand in a line with students whose hand spans are 10 cm $\leq w <$ 13 cm. At some appropriate time during the human histogram activity, each student should turn 90 degrees clockwise (or 90 degrees counterclockwise) so that everyone in a line within each interval is shoulder to shoulder. This gives students the opportunity to look at the overall shape of the histogram.

C Discuss how the graph's horizontal axis, interval widths, and so on, affect the shape of the histogram and what they should consider when determining the width of the histogram bars.

C Select a different interval width, lay down the correct numbers, and re-form a new human histogram.

C Ask students to tell the class in which ways box plots and histograms are the same (horizontal axis is scaled, rectangles are used, and so on) and different (box plot uses a five-number summary as its starting point, box plot divides the data into quartiles; intervals in a histogram are arbitrary, and so on.)

After working through the human versions of the graphs, go through the calculator procedure as outlined in Calculator Note 1D.

Note that the concept of variable is introduced in this lesson. Ask students for examples of variables that they have previously studied in this course. Some previously encountered variables are the pulse rates of their classmates in Investigation 1.1: Picturing Pulse Rates, the mint years of pennies in Investigation 1.2: Making "Cents" of the Center, and in Investigation 1.3: Pennies in a Box.

Investigation 7.5:
How Old Is That Artifact?

Did You Know?

In 2007, a reindeer herder in northwest Siberia, found the frozen carcass of a 6-month-old baby mammoth. Scientists believe the mammoth died at the end of the last Ice Age about 10,000 years ago. The baby's eyes and trunk were still intact, and the baby mammoth even still had fur on its body!

Reference: http://news.bbc.co.uk/2/hi/science/nature/6284214.stm

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You have probably watched or read news stories where they talk about fascinating ancient artifacts found by archaeologists. At an archaeological dig, objects such as stone tools, pottery, metal objects, buttons, jewelry, clothing, and bone fragments are often found. For example, you might hear that a piece of wooden tool has been unearthed and the archaeologist finds it to be 5,000 years old. You might learn that a child mummy was found high in the Andes and the archaeologist says the child lived more than 2,000 years ago. You might learn of the largest Tyrannosaurus Rex ever found that lived in what is now South Dakota, a dinosaur named Sue (named for the paleontologist who found it) and is believed to have lived between 60 and 70 million years ago.

How do scientists know how old an object or a bone fragment is? What methods do they use and how do these methods work?

Let's investigate how scientists use radioactivity to determine the age of objects by focusing on Carbon-14 dating. To determine the age of certain archaeological artifacts up to 50,000 years old, scientists often use Carbon-14 dating. However, scientists only use Carbon-14 dating to determine the age of objects that once were living, such as bone, cloth, wood, and plant remains. Carbon-12 and Carbon-14 are atoms found in all living organisms. As soon as a living organism dies, it stops taking in new Carbon. The ratio of Carbon-12 to Carbon-14 at the time of death is the same as the ratio of these atoms in all living things, but while the Carbon-12 remains, the Carbon-14 begins to decay and is not replaced.

HOW OLD IS AN ARTIFACT? (CONTINUED)

The rate at which Carbon-14 decays is predictable. Carbon-14 is an isotope that has a half-life of about 5700 years. This means that 5700 years after a living organism dies, it will have only half of the Carbon-14 it had at the time of its death. The rest will have decayed, and actually will have become Nitrogen-14 atoms. In the next 5700 years, it will lose half of what was remaining, so that after a living organism dies, the amount of Carbon-14 remaining 11,400 years later will be approximately one-fourth of the original amount. It continues to lose half of the remaining Carbon-14 every 5700 years. Meanwhile, the amount of Carbon-12 remains constant. By determining the ratio of Carbon-12 to Carbon-14 in a sample and comparing it to the ratio of a living organism, it is possible to determine the age of a formerly living organism to a reasonable degree of accuracy.

In this investigation, we will model the radioactive decay of an isotope such as Carbon-14, the process by which an unstable Carbon-14 atom loses energy by emitting radiation and decaying into a different type of atom.

Each group will need a paper plate and a cup of M&M's. The more M&M's you have, the better (for more than one reason!). Be careful! All of these M&M's are radioactive.

- Count the number of M&M's in the cup, and record this number as the amount of Carbon-14 atoms the organism has at the time of its death.
- Pour the M&M's on the paper plate. Remove all of the M&M's that have their M showing, and count them. Record this number in the table provided. This represents the number of Carbon-14 atoms that emitted radiation and became Nitrogen-14 atoms. These M&M's are no longer radioactive, and are now safe to eat if you choose.
- Next, put all of the radioactive M&M's back in the cup so you may again pour them onto the paper plate. Again, remove and count, all of the M&M's that have their M showing, and record this number. Continue this process until you have fewer than 5 radioactive M&M's left on the paper plate.

A Create a scatterplot of the number of radioactive M&M's as a function of the number of trials. Describe the graph.

B Can the number of radioactive M&M's with respect to the number of trials be modeled using an exponential function? Explain your reasoning.

C Write an algebraic equation to represent the number of radioactive M&M's that you would expect there to be as a function of the number of trials. What is the b-value of this exponential function? What is the value of a? What is the real-world meaning of both of these constants?

D Use regression to find an exponential function that models your real data.

E Compare your experimental equation with your theoretical equation.

F If the M&M's represent carbon-14 atoms, what does one trial represent? Explain.

Table for Investigation 7.5:
How Old Is That Artifact?

Use this table to record the number of M&M's that remain and those that are no longer radioactive ("M" showing) for each trial.

TRIAL NUMBER	# OF RADIOACTIVE M&M'S REMAINING	# OF M&M'S NO LONGER RADIOACTIVE
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

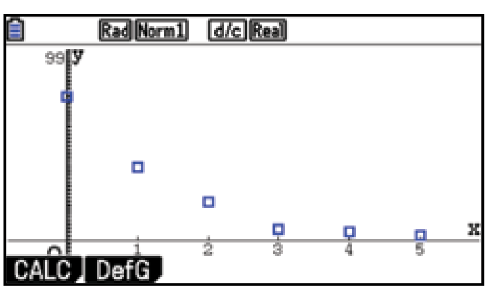
Sample Solution:
How Old Is That Artifact?

Results from one of our experiments are provided in the table below:

TRIAL NUMBER	# OF RADIOACTIVE M&M'S REMAINING	# OF M&M'S NO LONGER RADIOACTIVE
0	78	0
1	40	38
2	21	57
3	6	72
4	5	73
5	3	75
6	-	-
7	-	-
8	-	-
9	-	-
10	-	-

A Create a scatterplot of the number of radioactive M&M's as a function of the number of trials. Describe the graph.

Enter the Trial Number and the number of Radioactive M&M's remaining into two lists in the **Statistics** menu. Go to **GRAPH** and set up a scatterplot using the Trial Number as your **XList:** and the number of Radioactive M&M's remaining as your **YList:**. Note that we did not have that many trials before the number of M&M's was fewer than 5.



HOW OLD IS AN ARTIFACT? (CONTINUED)

We note that the graph is decreasing, that it is concave upward, meaning that it is decreasing at a slower and slower rate, and that it is solely in Quadrant 1.

B Can the number of radioactive M&M's with respect to the number of trials be modeled using an exponential function? Explain your reasoning.

Because we expect approximately half of the radioactive M&M's to remain after each trial, we have a geometric sequence. Whenever we have a constant multiplier (or a constant ratio between successive terms), we can model the situation with an exponential function. That is, a geometric sequence leads us to an exponential function.

C Write an algebraic equation to represent the number of radioactive M&M's that you would expect there to be as a function of the number of trials. What is the b-value of this exponential function? What is the value of a? What is the real-world meaning of both of these constants?

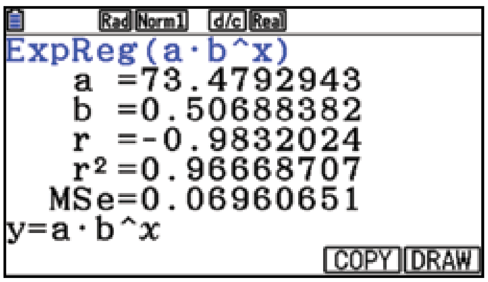
Let's assume we start with 78 M&M's. After 1 trial, we would expect 78 • 0.5 M&M's to still be radioactive. After 2 trials, we would expect 78 • 0.5 • 0.5 M&M's to be radioactive. For each additional trial, we would multiply the previous result by 0.5, which leads to the equation $y = 78 \cdot 0.5^x$, where x represents the number of trials and y represents the number of radioactive M&M's. We note, however, that our model cannot be perfect. Suppose 39 are left after the first trial, then we would expect 19.5 after the second trial. We know we can't have a partial M&M; either an M&M is radioactive or it isn't.

In the general form, $y = ab^x$, for our specific equation, a is 78, the number of M&M's with which we start, and b is 0.5, our multiplier, the factor we expect to have remaining after each trial.

D Use regression to find an exponential function that models your real data.

From the scatterplot as shown above:

- Press [F1] (CALC), [F6], and [F3] (EXP).
- Then press [F2] (ab^x) to choose the exponential form mentioned above.



Our result using regression is $y \approx 73.48 \cdot 0.51^x$.

E Compare your experimental equation with your theoretical equation.

In theory, our value for a should have been 78; we were not that far off the mark. Our value for b, in theory, should have been 0.50, and here we were even closer. However, students should not believe that they were wrong if their regression model does not come as close to their theoretical model as ours. Perhaps we were just having a lucky day. In any case, we believe it is worth class time exploring how much variability there is in the models from each group. Developing a sense of variability is essential for statistical literacy, something we believe is critical for all students.

F If the M&M's represent carbon-14 atoms, what does one trial represent? Explain.

One trial would represent 5,700 years, since we expect approximately half of graphically and with a table.

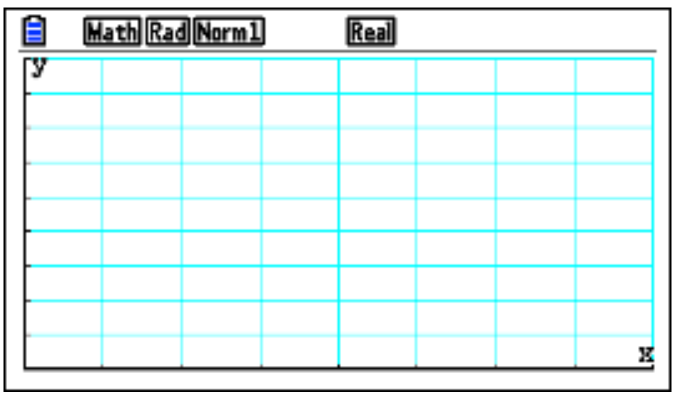
The Wave

Sometimes at sporting events, people in the audience stand up quickly in succession with their arms upraised and then sit down again. The continuous rolling motion that this creates through the crowd is called "the wave". You and your class will investigate how long it takes different-size groups to do the wave.

1. Using different-size groups, determine the time for each group to complete the wave. Collect at least nine pieces of data in the form (*number of people, time*) and record them in this table.

Number of people	Time (seconds)

2. Plot the points, and find the equation of a reasonable line of fit.



Q1 What is the slope of your line, and what is its real-world meaning?

Q2 What are the *x*- and *y*-intercepts of your line, and what are their real-world meanings?

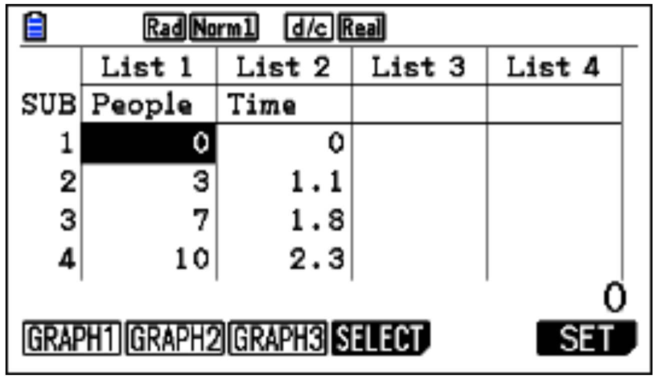
Q3 What is a reasonable domain for this function? Why?

Q4 Can you use your line of fit to predict how long it would take to complete the wave if everyone at your school participated? Everyone in a large stadium? Explain why or why not.

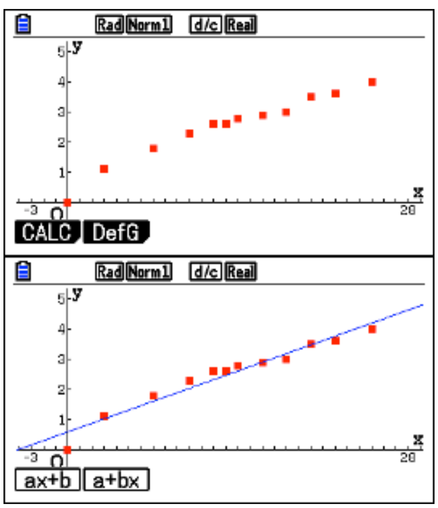
Adapted from Discovering Advanced Algebra, 2nd Edition, Kendall Hunt Publishing

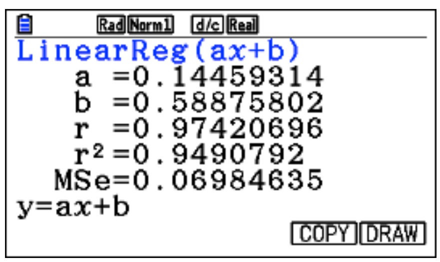
1. Answers may vary. Once data is collected, transfer into List 1 and List 2 in the Statistics mode.

Number of people	Time (seconds)
0	0
3	1.1
7	1.8
10	2.3
12	2.6
13	2.3
14	2.8
16	2.9
18	3
20	3.5
22	3.6
25	4



2. Answers may vary, depending on the plotted points.





Q1 The slope of the line is 0.14459314. In this problem, it will take each person about .14 seconds to complete the process of raising their hands.

Q2 The *x*- and *y*-coordinates represent the time in seconds (*y*-coordinates) that it would take for a certain number of people (*x*-coordinates) to participate in the wave. In this problem, the *x*-intercept would be the number of people before the wave began ($x = 0$) and the *y*-intercept would ideally be zero as well. In this data set, the *y*-intercept is 0.5887502 seconds.

Adapted from Discovering Advanced Algebra, 2nd Edition, Kendall Hunt Publishing

Q3 A reasonable domain for The Wave problem would be the set of whole numbers because the independent variable in this problem is the number of people participating in the wave. Assume the number of people cannot be negative, rational, or irrational!

Q4 Answers may vary. The line of fit could be used to make a prediction (the *r* value is .97), but the larger the number of people participating is, the less accurate the prediction will be.