which has general solution

$$I(a) = c_1 e^{ab} + c_2 e^{-ab}.$$

Letting $a \to 0$ shows that

$$I(0) = c_1 + c_2 = \int_0^\infty \frac{1}{x^2 + b^2}\, dx = \left[\frac{1}{b} \tan^{-1} \frac{x}{b}\right]_0^\infty = \frac{\pi}{2b}.$$

Also, from the alternative form for $I(a)$, as $a \to \infty$ $I(a) \to 0$. Hence $c_1 = 0$ and $c_2 = \pi/2b$ giving the value of the integral as

$$I(a) = \int_0^\infty \frac{\cos ax}{x^2 + b^2}\, dx = \frac{\pi e^{-ab}}{2b}.$$

Based on notes and correspondence from

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73.46 One parabola or many?

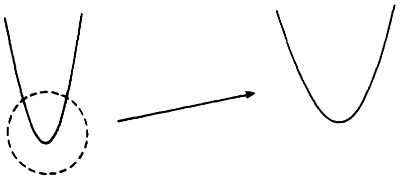
The following makes an interesting problem for sixth forms:

There is only *one* circle shape.
(All circles can be got from one circle by enlargement.)
But there are *many* different shaped ellipses.

There is only *one* square shape.
(All squares can be got from one square by enlargement.)
But there are *many* different shaped rectangles.

Which category does the parabola come into?
Are there *many* parabolas, of different shapes?
Or is there only *one* parabola shape?
Prove your answer, whichever it is.

It usually leads to heated arguments. Some people observe that the point of a sharp needle under a microscope looks like a blunt needle:



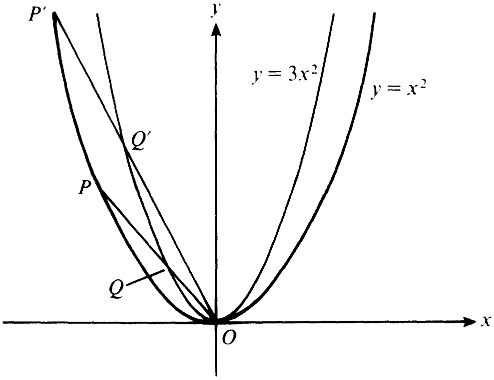
and eventually the conclusion is reached that there is just one parabolic shape.

In general the parabola $y = kx^2$ ($k > 0$) when enlarged by a scale factor of k gives the parabola $y = x^2$:

$$y = kx^2$$
$$\downarrow \quad \downarrow$$
$$\frac{y}{k} = k\left(\frac{x}{k}\right)^2 \quad \text{or} \quad y = x^2$$

e.g.

$$\frac{OP}{OQ} = \frac{OP'}{OQ'} = 3$$



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73.47 The four points on a parabola and circle

Suppose that a circle intersects a parabola, given parametrically as $(a\lambda^2, 2a\lambda)$, at four points with parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. It is easy to prove (and very familiar to most elderly mathematicians) that $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0$. What may be less familar is the corresponding result for the natural parameters of the circle.